Module 4-2
Methods of Quantitative Reliability Analysis

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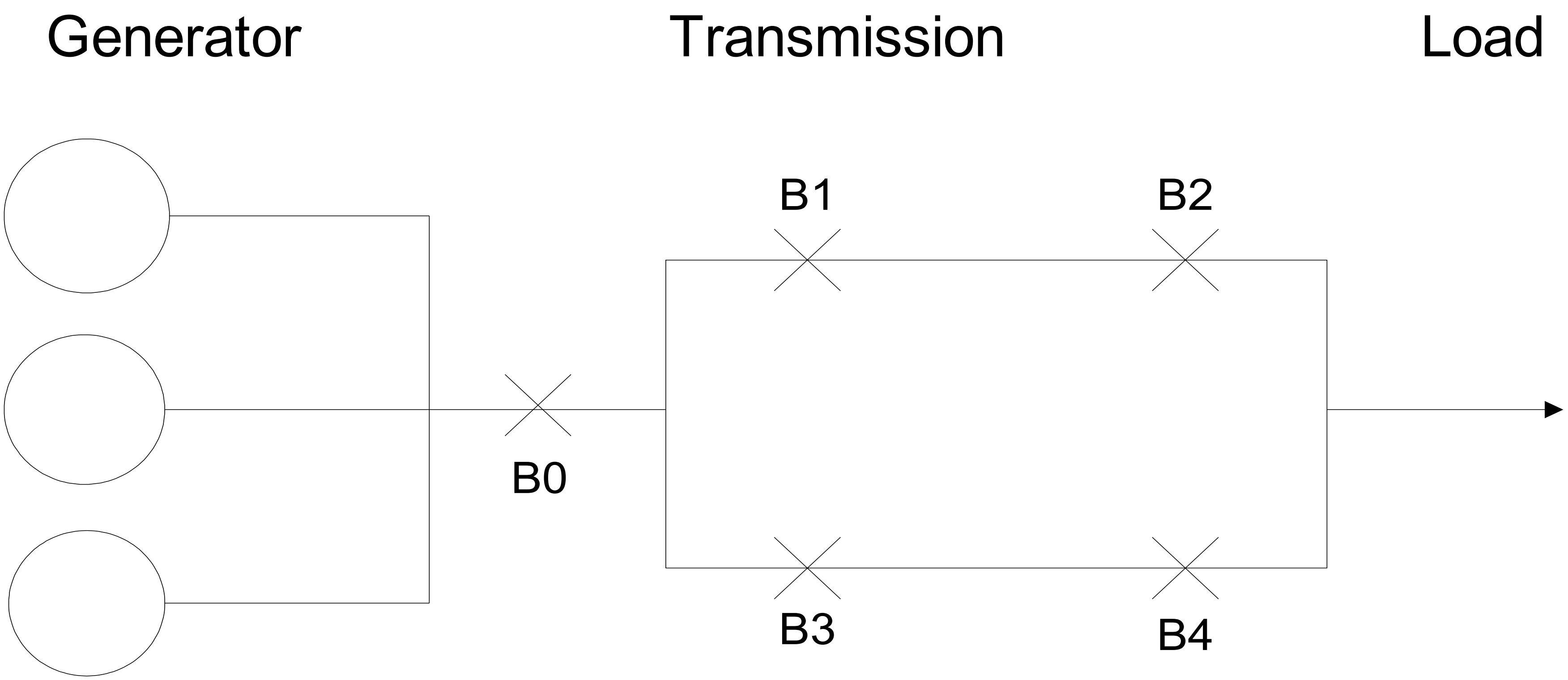
METHODS OF QUANTITATIVE RELIABILITY ANALYSIS

- ANALYTICAL METHODS
  - STATE SPACE USING MARKOV PROCESSES
  - NETWORK REDUCTION
  - MIN CUT SETS

- MONTE CARLO SIMULATION
  - NONSEQUENTAL - RANDOM SAMPLING
  - TIME SEQUENTIAL

- CONCEPT OF RELIABILITY COHERENCE

EXAMPLE SYSTEM



EXAMPLE SYSTEM

- Generators:

Each generator either has full capacity of 50 MW or 0 MW when failed. Failure rate of each generator is 0.1/day and mean-repair-time is 12 hours

- Transmission Lines:

The failure rate of each transmission line is assumed to be 10 f/y during the normal weather and 100 f/y during the adverse weather. The mean down time is 8 hours. Capacity of each line is 100 MW.

- Weather:

The weather fluctuates between normal and adverse state with mean duration of normal state 200 hours and that of adverse state 6 hours.

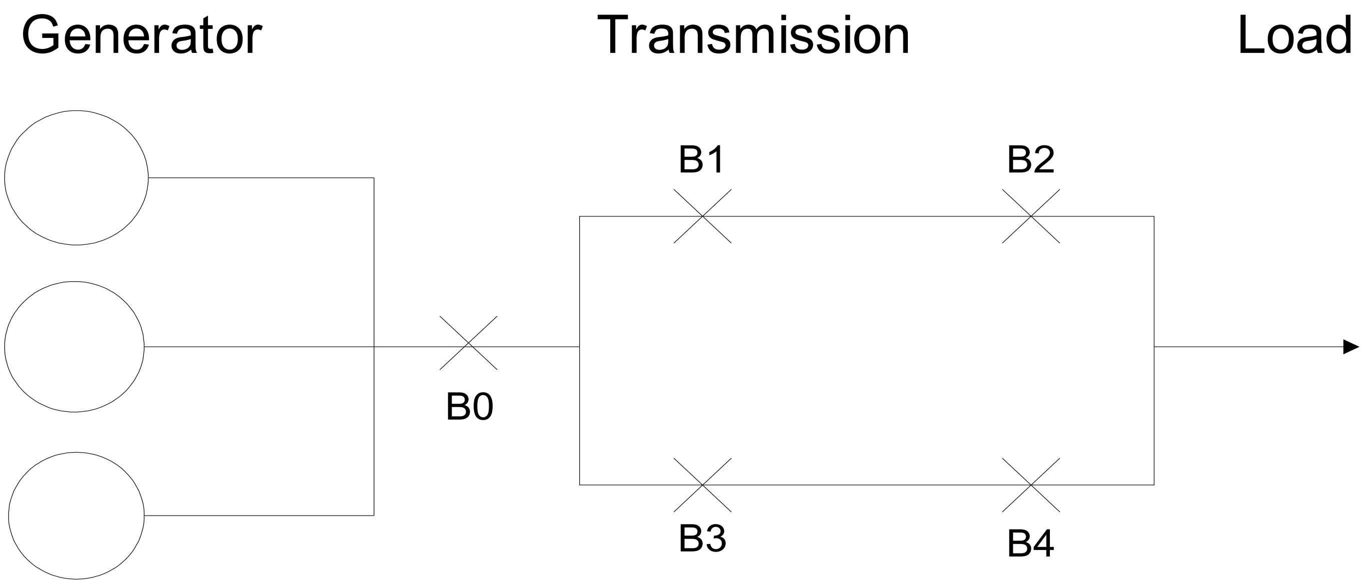
- Breakers:

Breakers are assumed perfectly reliable except that the pair B1&B2 or B3&B4 may not open on fault on the transmission line with probability 0.1.

- Load:

Load fluctuates between two states, 140 MW and 50 MW with mean duration in each state of 8hr and 16hr respectively.

EXAMPLE SYSTEM



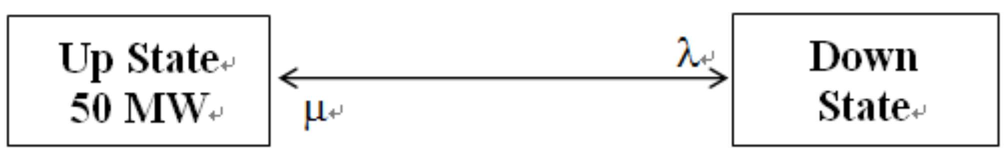
- FOR THE DESCRIBED SYSTEM, HOW CAN YOU CALCULATE THE FOLLOWING BASIC RELIABILITY INDICES ?

1. Loss of load probability
2. Frequency of loss of load
3. Mean duration of loss of load

Solving Example Problem using Markov Approach

- 1. System State Description & Equivalents

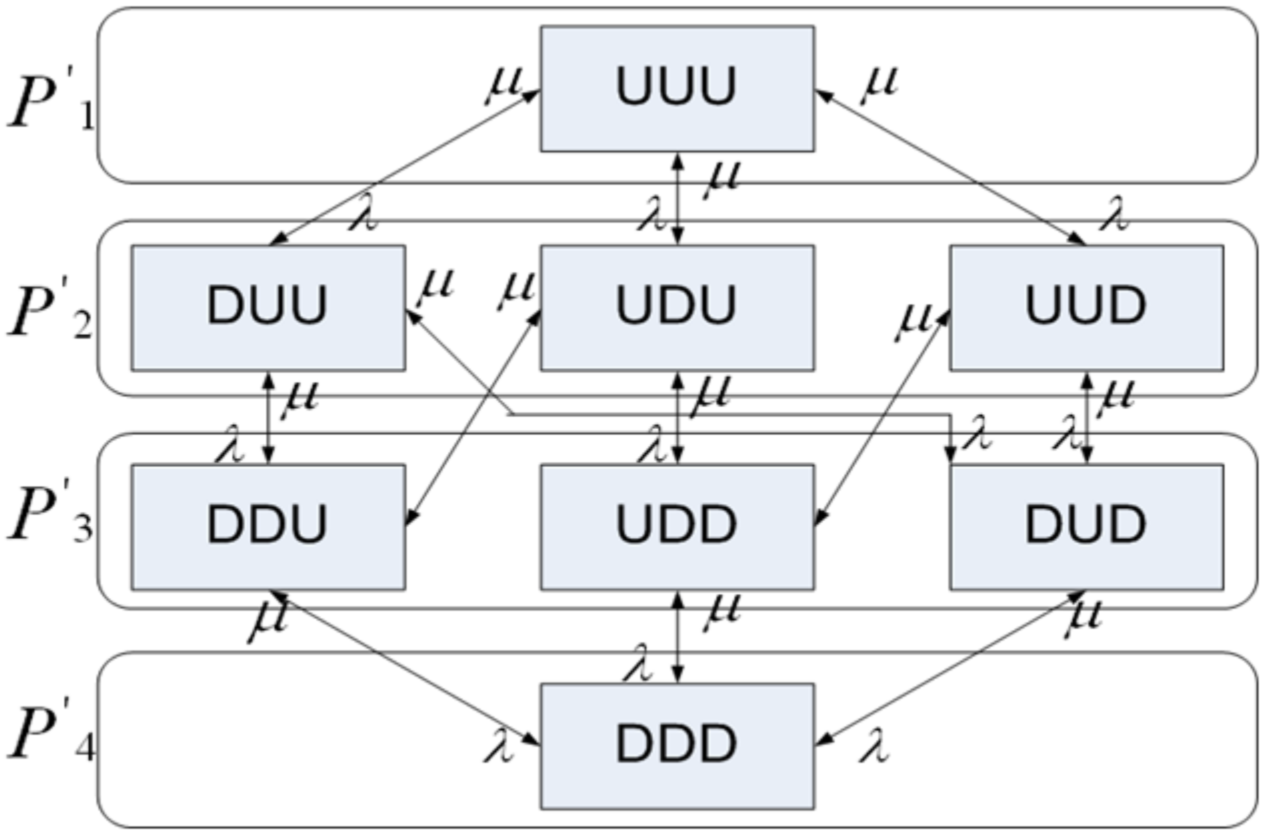
- The first task is to obtain probabilities for the generators, transmission lines and loads, which are independent parts of the system.
- 1. 1. Generators



Each Generator has two possible states

$$\mu = \frac{1}{\frac{12}{8760}} = 730/\,year \qquad \lambda = 0.1/\text{day} = 36.5/\text{year}$$

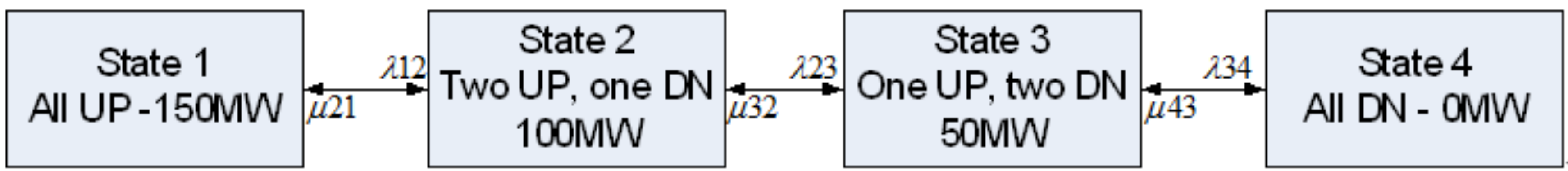
Solving Example Problem using Markov Approach



State Transition Diagram – Generator System

Solving Example Problem using Markov Approach

- Merging Identical Capacity States



Equivalent State Transition Diagram – Generator System

- Equivalent transition rates:

$$\lambda_{12G} = \frac{P_1\lambda + P_1\lambda + P_1\lambda}{P_1} = 3\lambda$$

$$\mu_{21G} = \frac{P_2\mu + P_3\mu + P_4\mu}{P_2 + P_3 + P_4} = \mu$$

$$\lambda_{23G} = \frac{2P_2\lambda + 2P_3\lambda + 2P_4\lambda}{P_2 + P_3 + P_4} = 2\lambda$$

$$\mu_{32G} = \frac{2P_5\mu + 2P_6\mu + 2P_7\mu}{P_5 + P_6 + P_7} = 2\mu$$

$$\lambda_{34G} = \frac{P_5\lambda + P_6\lambda + P_7\lambda}{P_5 + P_6 + P_7} = \lambda$$

$$\mu_{43G} = \frac{P_8\mu + P_8\mu + P_8\mu}{P_8} = 3\mu$$

Solving Example Problem using Markov Approach

- Transition rate matrix is:

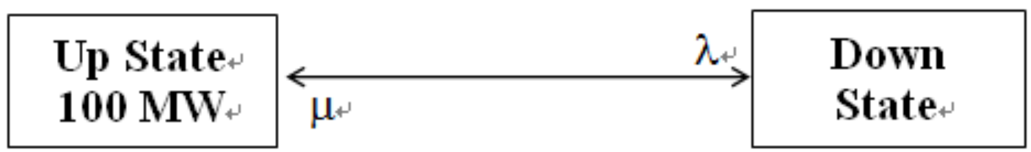
$$R_G = \begin{bmatrix} -3\lambda & 3\lambda & 0 & 0 \\ \mu & -(\mu + 2\lambda) & 2\lambda & 0 \\ 0 & 2\mu & -(2\mu + \lambda) & \lambda \\ 0 & 0 & 3\mu & -3\mu \end{bmatrix}$$

- If we substitute values for μ and λ obtained in the beginning into above matrix, transition rate matrix for the generator system is:

$$R_G = \begin{bmatrix} -109.5 & 109.5 & 0 & 0 \\ 730 & -803 & 73 & 0 \\ 0 & 1460 & -1496.5 & 36.5 \\ 0 & 0 & 2190 & -2190 \end{bmatrix}$$

Solving Example Problem using Markov Approach

- 1. 2. Transmission Lines



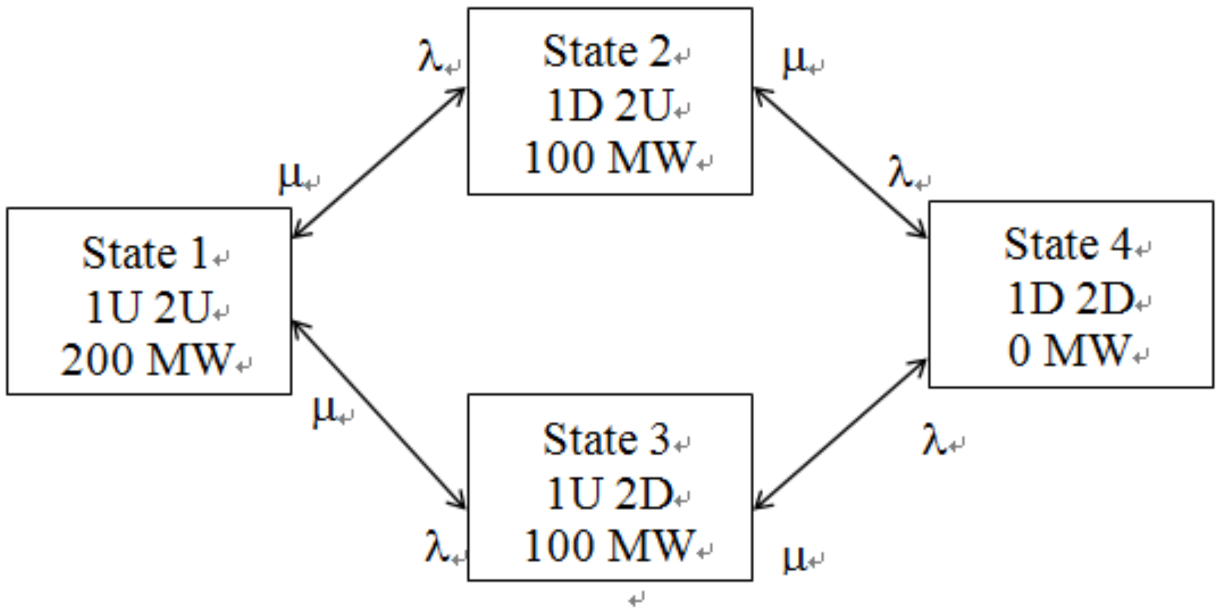
Each Transmission line has two possible states

- During the normal weather $\lambda = 10/\text{year}$
- During the adverse weather $\lambda' = 100/\,year$

$$\mu = \frac{1}{\frac{8}{8760}} = 1095/\text{year}$$

Solving Example Problem using Markov Approach

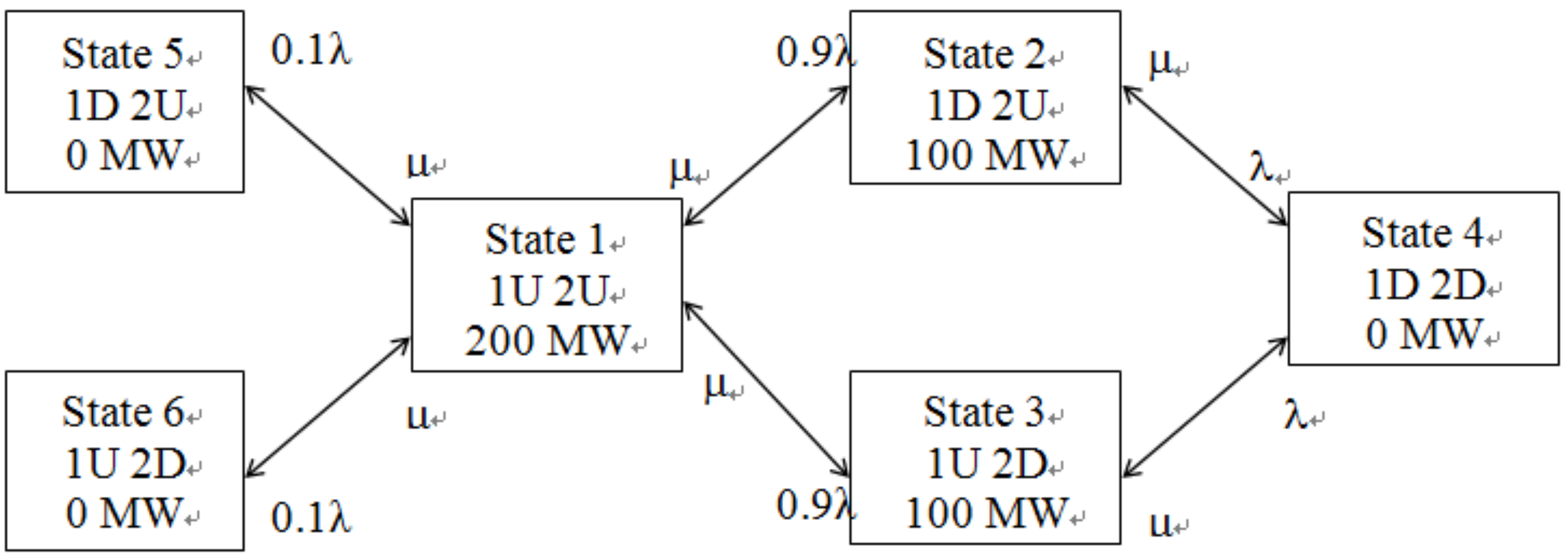
- If all the breakers are perfectly reliable, for the two-transmission-line system, there will be 4 states.



Four State Transition Diagram – Transmission System

Solving Example Problem using Markov Approach

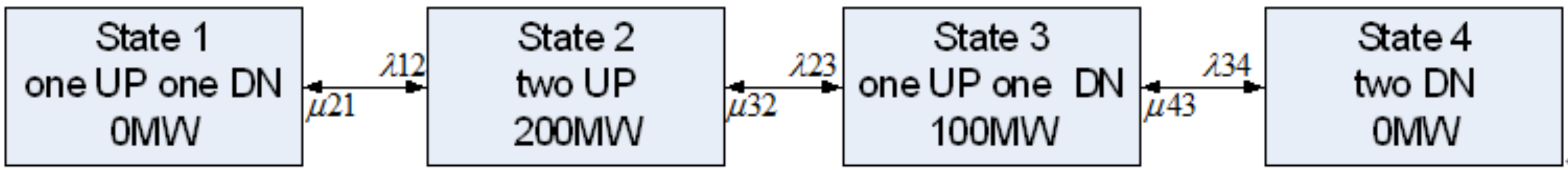
- If breakers may not open on command:



Six State Transition Diagram – Transmission System

Solving Example Problem using Markov Approach

- Merging of states:



Equivalent Four State Transition Diagram – Transmission System

- Equivalent transition rates:

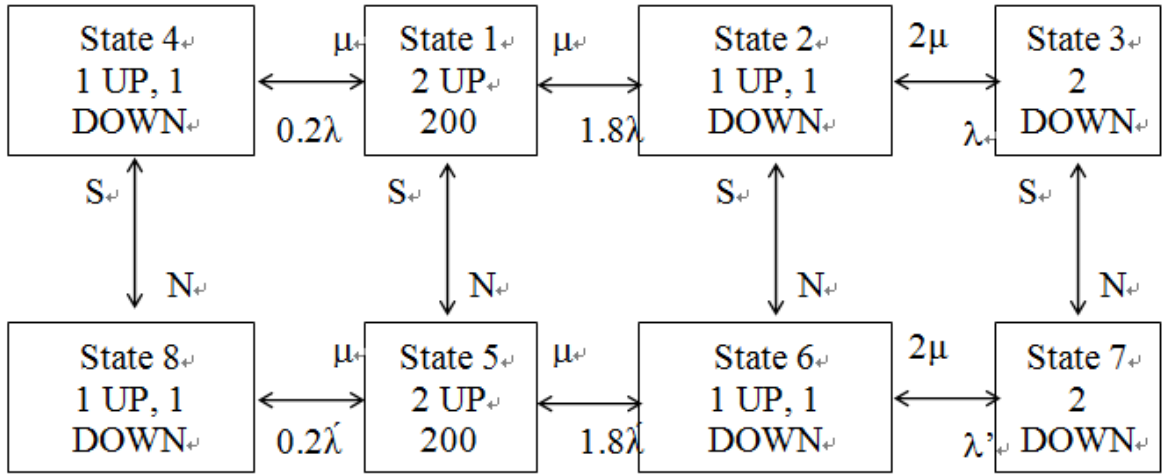
$$\lambda_{12} = \frac{P_5\mu + P_6\mu}{P_5 + P_6} = \mu \qquad \mu_{21} = \frac{P_1(0.1\lambda + 0.1\lambda)}{P_1} = 0.2\lambda$$

$$\lambda_{23} = \frac{P_1(0.9\lambda + 0.9\lambda)}{P_1} = 1.8\lambda \qquad \mu_{32} = \frac{P_2\mu + P_3\mu}{P_2 + P_3} = \mu$$

$$\lambda_{34} = \frac{P_2\lambda + P_3\lambda}{P_2 + P_3} = \lambda \qquad \mu_{43} = \frac{P_4(\mu + \mu)}{P_4} = 2\mu$$

Solving Example Problem using Markov Approach

- 1.2.1 Weather



Equivalent Eight State Transition Diagram – Transmission System

- Transition rate from normal weather to adverse weather is $N = \frac{1}{\frac{200}{8760}} = 43.8/\text{year}$
- Transition rate from adverse weather to normal weather is: $S = \frac{1}{\frac{6}{8760}} = 1460/year$

Solving Example Problem using Markov Approach

- Transition rate matrix of transmission system is:

$$
R_T = \begin{bmatrix}
-(2\lambda + N) & 1.8\lambda & 0 & 0.2\lambda & N & 0 & 0 & 0 \\
\mu & -(\mu + \lambda + N) & \lambda & 0 & 0 & N & 0 & 0 \\
0 & 2\mu & -(2\mu + N) & 0 & 0 & 0 & N & 0 \\
\mu & 0 & 0 & -(\mu + N) & 0 & 0 & 0 & N \\
S & 0 & 0 & 0 & -(2\lambda' + S) & 1.8\lambda' & 0 & 0.2\lambda' \\
0 & S & 0 & 0 & \mu & -(\mu + \lambda' + S) & \lambda' & 0 \\
0 & 0 & S & 0 & 0 & 2\mu & -(2\mu + S) & 0 \\
0 & 0 & 0 & S & \mu & 0 & 0 & -(\mu + S)
\end{bmatrix}
$$

Solving Example Problem using Markov Approach

- 1.3 Load

State Transition Diagram – Load

$$\lambda_{21} = \frac{1}{\frac{16}{8760}} = 547.5 / year \qquad \lambda_{12} = \frac{1}{\frac{8}{8760}} = 1095 / year$$

- Transition rate matrix:

$$R_L = \begin{bmatrix} -1095 & 1095 \\ 547.5 & -547.5 \end{bmatrix}$$

Solving Example Problem using Markov Approach

- 2 Steady State Probabilities, Frequency and Mean Duration of Loss of Load
- 2. 1. Generation System
- In order to get the steady probability of each state, we can write:

$$R_G^T \begin{bmatrix} P_{1G} \\ P_{2G} \\ P_{3G} \\ P_{4G} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \sum_{i=1}^{4} P_{iG} = 1$$

- Using the RG we obtained to solve above equation, we get the steady state probability of each state.
- If generators are independent probabilities can be calculated by product rule also.
- Probabilities calculated in either way are the same.

$$P_u = \frac{\mu}{\lambda + \mu} = 0.95238$$

$$P_d = \frac{\lambda}{\lambda + \mu} = 0.047619$$

$$P_{1G} = P_u * P_u * P_u = 0.8638377$$

$$P_{2G} = 3 * P_u * P_u * P_d = 0.1295725$$

$$P_{3G} = 3 * P_u * P_d * P_d = 0.00647876$$

$$P_{4G} = P_d * P_d * P_d = 0.000107979$$

Solving Example Problem using Markov Approach

- 2. 2. Transmission System
- We have the following equations:

$$R_T^T \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \sum_{i=1}^{8} P_i = 1$$

Solving Example Problem using Markov Approach

- Using the RT we obtained to solve previous equation, we get the steady state probability of each state:

R_t^T =

1.0e+003 *

-0.0638	1.0950	0	1.0950	1.4600	0	0	0
0.0180	-1.1488	2.1900	0	0	1.4600	0	0
0	0.0100	-2.2338	0	0	0	1.4600	0
0.0020	0	0	-1.1388	0	0	0	1.4600
0.0438	0	0	0	-1.6600	1.0950	0	1.0950
0	0.0438	0	0	0.1800	-2.6550	2.1900	0
0	0	0.0438	0	0	0.1000	-3.6500	0
0	0	0	0.0438	0.0200	0	0	-2.5550

P_1=0.9507726

P_2=0.01787034

P_3=0.0001196528

P_4=0.0019820304

P_5=0.02678843

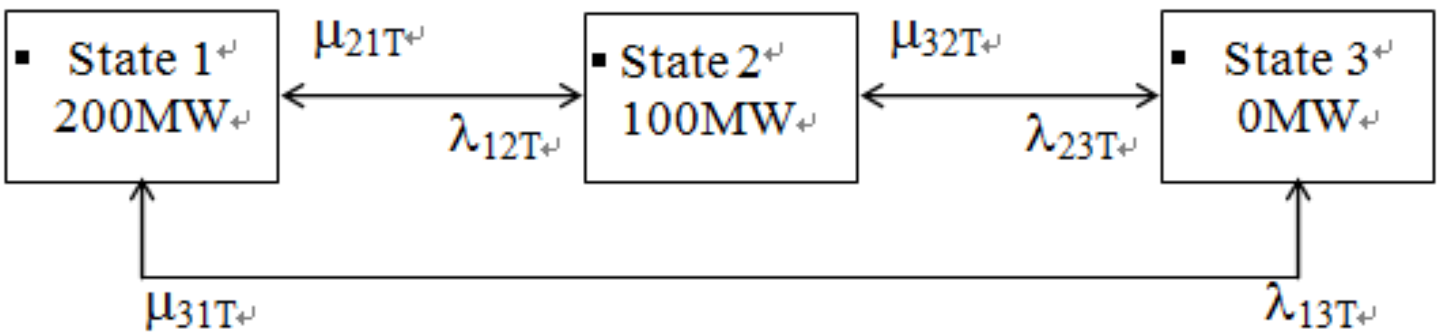
P_6=0.002162378

P_7=0.000060686

P_8=0.00024383

Solving Example Problem using Markov Approach

- We can also reduce the eight-state transmission transition diagram to a three-state diagram with respect to the capacities of the states:



Equivalent Three State Transition Diagram – Transmission System

Solving Example Problem using Markov Approach

- For the reduced model, the following results apply:

$$P_{1T} = P_1' + P_5' = 0.9507726 + 0.02678843 = 0.97756103$$

$$P_{2T} = P_2' + P_6' = 0.01787034 + 0.002162378 = 0.020032718$$

$$P_{3T} = P_3' + P_4' + P_7' + P_8' = 0.0001196528 + 0.0019820304 + 0.000060686 + 0.00024383$$

$$P_{3T} = P_3' + P_4' + P_7' + P_8' = 0.0024061992$$

$$\lambda_{12T} = \frac{P_1' \cdot 1.8\lambda + P_5' \cdot 1.8\lambda'}{P_1' + P_5'} = \frac{0.9507726 \cdot 18 + 0.02678843 \cdot 180}{0.97756103} = 22.439339$$

$$\mu_{21\mathrm{T}} = \frac{P_2'\mu + P_6'\mu}{P_2' + P_6'} = \mu = 10$$

$$\lambda_{23T} = \frac{P_2'\lambda + P_6'\lambda'}{P_2' + P_6'} = \frac{0.01787034 \cdot 10 + 0.002162378 \cdot 100}{0.020032718} = 19.714808$$

$$\mu_{32\mathrm{T}} = \frac{P_3' \cdot 2\mu + P_7' \cdot 2\mu}{P_3' + P_7' + P_4' + P_8'} = 1095$$

$$\lambda_{13T} = \frac{P_1' \cdot 0.2\lambda + P_5' \cdot 0.2\lambda'}{P_1' + P_5'} = \frac{0.9507726 \cdot 2 + 0.02678843 \cdot 20}{0.97756103} = 2.493259$$

$$\mu_{31\mathrm{T}} = \frac{P_4'\mu + P_8'\mu}{P_3' + P_7' + P_4' + P_8'} = 2090$$

Solving Example Problem using Markov Approach

- 2. 3. Load
- The following equations apply:

$$R_L^T \begin{bmatrix} P_{1L} \\ P_{2L} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \sum_{i=1}^{2} P_{iL} = 1$$

- Using the RL we obtained to solve above equations we get the steady state probability in each state:

$$\begin{bmatrix} P_{1L} \\ P_{2L} \end{bmatrix} = \begin{bmatrix} 0.3333333 \\ 0.6666667 \end{bmatrix}$$

Solving Example Problem using Markov Approach

- 2. 4. Solution for the System

- Steady state probability, frequency and mean time of loss of load could be found using the following table:

$P_{1G} = 0.8638377$	$P_{1T} = 0.97756103$	$P_{1L} = 0.3333333$
$P_{2G} = 0.1295725$	$P_{2T} = 0.020032718$	$P_{2L} = 0.6666667$
$P_{3G} = 0.00647876$	$P_{3T} = 0.0024061992$	
$P_{4G} = 0.000107979$		

Solving Example Problem using Markov Approach

System State	Generation, transmission, load system state	Probability of system state	Transition to the states with loss of load	Loss of load
1	111	0.281485	2,3,4 ($\lambda_{12T}, \lambda_{13T}, \lambda_{12G}$)	No
2	121			Yes
3	131			Yes
4	211			Yes
5	221			Yes
6	231			Yes
7	311			Yes
8	321			Yes
9	331			Yes
10	411			Yes
11	421			Yes
12	431			Yes
13	112	0.562969	15(λ_{13T})	No
14	122	0.0115367	2,15 ($\lambda_{21L}, \lambda_{23T}$)	No
15	132			Yes
16	212	0.0844453	4,18 ($\lambda_{21L}, \lambda_{13T}$)	No
17	222	0.0017305	5,18 ($\lambda_{21L}, \lambda_{23T}$)	No
18	232			Yes
19	312	0.00422226	7,21,22, ($\lambda_{21L}, \lambda_{13T}, \lambda_{34G}$)	No
20	322	0.000086525	8,21,23 ($\lambda_{21L}, \lambda_{23T}, \lambda_{34G}$)	No
21	332			Yes
22	412			Yes
23	422			Yes
24	432			Yes

Solving Example Problem using Markov Approach

- We can calculate the probability of states having no load loss. Those probabilities are obtained for the generators, transmission lines and loads as independent.

- From previous Table , we can get the steady state probability of the loss of load as follows.

P=1-(0.281485+0.562969+0.0115367+0.0844453+0.0017305+0.00422226+0.000086525)

P=0.053524715

Solving Example Problem using Markov Approach

- The frequency of loss of load is:

$$\mathrm{F} = \sum_{i \in X^+} \sum_{j \in X^-} P_i \lambda_{ij} = \mathrm{P}_1(\lambda_{12\mathrm{T}} + \lambda_{13\mathrm{T}} + \lambda_{12\mathrm{G}}) + \mathrm{P}_{13}\lambda_{13\mathrm{T}} + \mathrm{P}_{14}(\lambda_{21\mathrm{L}} + \lambda_{23\mathrm{T}})$$

$$+ P_{16}(\lambda_{21L} + \lambda_{13T}) + P_{17}(\lambda_{21L} + \lambda_{23T}) + P_{19}(\lambda_{21L} + \lambda_{13T} + \lambda_{34G}) + P_{20}(\lambda_{21L} + \lambda_{23T} + \lambda_{34G})$$

F = 95.742635 /year

- Values needed for F that are calculated previously:

$$\lambda_{12T} = 22.439339 \qquad \lambda_{23T} = 19.714808 \qquad \lambda_{13T} = 2.493259$$

$$\lambda_{12G} = 3\lambda = 109.5$$

$$\lambda_{23G} = 2\lambda = 73 \qquad \lambda_{21L} = 547.5 / year$$

$$\lambda_{34G} = \lambda = 36.5$$

Solving Example Problem using Markov Approach

- The mean time of loss of load is:

$$MD = \frac{P}{F} = \frac{0.053524715}{95.742635 / year} = 4.89726 hours$$

Cut Set Method

- A cut set is a set of components or conditions that cause system failure.
  - A min cut set is a cut set that does not contain any cut set as a subset.
  - In this presentation a cut set implies a min cut set.
  - The term component will be used to indicate both a physical component as well as a condition.
- Components in a given cut set are in parallel, as they all need to fail to cause system failure.
- Cut sets are in series as any cut set can cause system failure.

Frequency & Duration Equations For Cut Sets

- First Order Cut Set: One component involved

$$\lambda_{csk} = \lambda_i \qquad r_{csk} = r_i$$

where

$\lambda_i, r_i =$ Failure rate and mean duration of component i

$\lambda_{csk}, r_{csk} =$ Failure rate and mean duration of cut set k that contains component i

Frequency & Duration Equations For Cut Sets

- Second Order Cut Set k: Two components involved

$$\lambda_{csk} = \frac{\lambda_i \lambda_j (r_i + r_j)}{1 + \lambda_i r_i + \lambda_j r_j} \qquad r_{csk} = \frac{r_i r_j}{r_i + r_j}$$

where

$\lambda_i, \lambda_j =$ Failure rates of components i and j comprising cut set k

$r_i, r_j =$ Mean failure durations of components i and j comprising cut set k

Frequency & Duration Equations For Cut Sets

- Second Order Cut Set with Components subject to Normal and Adverse Weather.

$\lambda_i, \lambda_i' =$ Failure rate of component i in the normal and adverse weather

$N, S =$ Mean duration of normal and adverse weather

$$\lambda_{csk} = \lambda_a + \lambda_b + \lambda_c + \lambda_d$$

$$\lambda_a = \frac{N}{N+S}\left(\frac{\lambda_i \lambda_j N r_i}{N + r_i} + \frac{\lambda_j \lambda_i N r_j}{N + r_j}\right)$$

$$\lambda_b = \frac{N}{N+S}\left(\lambda_i \frac{r_i}{N} \lambda_j' \frac{S r_i}{S + r_i} + \lambda_j \frac{r_j}{N} \lambda_i' \frac{S r_j}{S + r_j}\right)$$

$$\lambda_c = \frac{S}{N+S}\left(\lambda_i' \lambda_j \frac{N r_i}{N + r_i} + \lambda_j' \lambda_i \frac{N r_j}{N + r_j}\right)$$

$$\lambda_d = \frac{S}{N+S}\left(\lambda_i' \lambda_j' \frac{S r_i}{S + r_i} + \lambda_j' \lambda_i' \frac{S r_j}{S + r_j}\right)$$

$\lambda_a =$ Component due to both failures occurring during normal weather

$\lambda_b =$ Initial failure in normal weather, second failure in adverse weather

$\lambda_c =$ Initial failure in adverse weather, second failure in normal weather

$\lambda_d =$ Both failures during adverse weather.

Combining n Cut Sets

$$\lambda_T = \lambda_{cs1} + \lambda_{cs2} + \cdots + \lambda_{csn}$$

$$r_T = (\lambda_{cs1} r_{cs1} + \lambda_{cs2} r_{cs2} + \cdots + \lambda_{csn} r_{csn}) / \lambda_T$$

APPLICATION OF CUT SET METHOD TO EXAMPLE SYSTEM

- Cut set 1: One line failure and breaker stuck.

$$\lambda_{av} = \frac{\lambda N + \lambda' S}{N + S} = 12.621$$

$$\lambda_{cs1} = 2 \times \lambda_{av} \times 0.1 = 2.524 f / y$$

$$r_{cs1} = 8hr$$

- Cut set 2: One generator failure and load changes from 50 to 140

$$\lambda_g = 0.1 / day = 36.5 / year$$

$$\lambda_{load} = \frac{8760}{16} = 547.5 / year$$

$$r_g = 12hr = .00137 yr$$

$$r_{load} = 8hr = .000913 yr$$

$$\lambda_{cs2} = \frac{\lambda_g \lambda_{load} (r_g + r_{load})}{(1 + \lambda_g r_g + \lambda_{load} r_{load})} = 88.306 / yr$$

$$r_{cs2} = \frac{r_g r_{load}}{r_g + r_{load}} = 4.8hr$$

APPLICATION OF CUT SET METHOD TO EXAMPLE SYSTEM

- Cut set 3:One line failure (breaker not stuck) and load changes from 50 to 140.

$$\lambda_l = \lambda_{av} \times 0.9$$
$$r_l = 8hr$$

$$\lambda_{cs3} = \frac{\lambda_l \lambda_{load}(r_l + r_{load})}{(1 + \lambda_l r_l + \lambda_{load} r_{load})} = 15.03 / yr$$

$$r_{cs3} = \frac{r_l r_{load}}{r_l + r_{load}} = 4hr$$

- Cut set 4: Two lines fail(breaker not stuck)
- For each line

$$\lambda = 10 \times .9 = 9 / yr$$
$$\lambda' = 100 \times .9 = 90 / yr$$
$$r = 8hr = .000913 yr$$
$$N = 200hr = .022831 yr$$
$$S = 6hr = .000685 yr$$

- Applying the equation for second order cut set exposed to fluctuating environment,

$$\lambda_{cs4} = .3888 / yr$$
$$r_{cs4} = 4hr.$$

APPLICATION OF CUT SET METHOD TO EXAMPLE SYSTEM

- Cut set 4: Two lines fail(breaker not stuck)
- For each line

$$\lambda = 10 \times .9 = 9 / yr$$
$$\lambda' = 100 \times .9 = 90 / yr$$
$$r = 8hr = .000913 yr$$
$$N = 200hr = .022831 yr$$
$$S = 6hr = .000685 yr$$

- Applying the equation for second order cut set exposed to fluctuating environment,

$$\lambda_{cs4} = .3888 / yr$$
$$r_{cs4} = 4hr.$$

- For the system

$$\lambda_T = \lambda_{cs1} + \lambda_{cs2} + \lambda_{cs3} + \lambda_{cs4} = 106.25 / yr$$
$$r_T = \frac{(\lambda_{cs1} r_{cs1} + \lambda_{cs2} r_{cs2} + \lambda_{cs3} r_{cs3} + \lambda_{cs4} r_{cs4})}{\lambda_T} = 4.76hr$$
$$\mu_T = \frac{1}{r_T}$$

$$\text{Frequency of failure} = \frac{\mu_T}{\lambda_T + \mu_T} \lambda_T = 100.45 / yr$$

$$\text{Probability of failure} = \frac{\lambda_T}{\lambda_T + \mu_T} = .0546$$