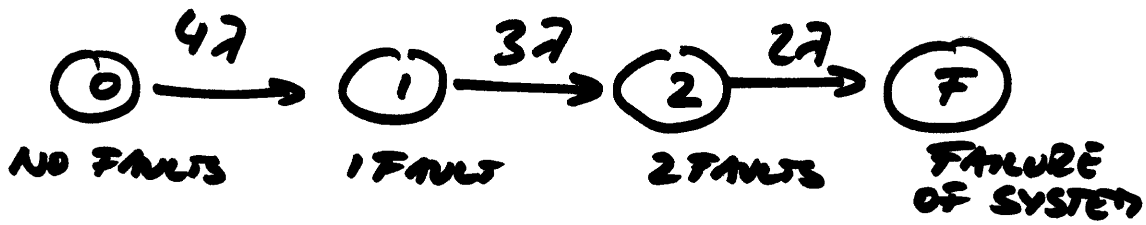
EXAMPLE 2:

SYSTEM WITH 4 MODULES.
2 FAILURES CAN BE TOLERATED.

ASSUME: EACH MODULE FAILS WITH RATE "λ".



$$\frac{dP_0}{dt} = -4\lambda P_0(t)$$

$$\frac{dP_1}{dt} = +4\lambda P_0(t) - 3\lambda P_1(t)$$

$$\frac{dP_2}{dt} = 3\lambda P_1(t) - 2\lambda P_2(t)$$

$$\frac{dP_F}{dt} = 2\lambda P_2(t)$$

SOLUTION FOR EXAMPLE 2:

$$P_0(t) = e^{-4\lambda t}$$

$$P_1(t) = 4e^{-3\lambda t}(1-e^{-\lambda t})$$

$$P_2(t) = 6e^{-2\lambda t}(1-e^{-\lambda t})^2$$

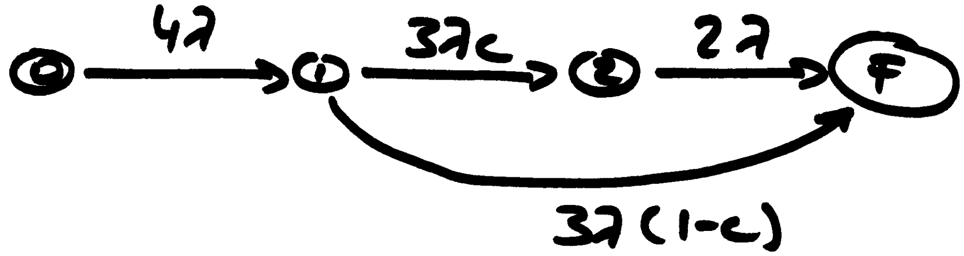
$$P_F(t) = 1 - P_0(t) - P_1(t) - P_2(t)$$

$$T = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ F \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & F \end{array} \\ \begin{bmatrix} -4\lambda & 0 & 0 & 0 \\ 4\lambda & -3\lambda & 0 & 0 \\ 0 & 3\lambda & -2\lambda & 0 \\ 0 & 0 & 2\lambda & 0 \end{bmatrix} \end{array}$$

SOLVE THE SYSTEM OF diff. Eq USING LAPLACE TRANSFORM

$$[sI - T]^{-1} \quad \ldots\ldots \text{etc}\ldots$$

MODELING COVERAGE



$$\frac{dP_0}{dt} = -4\lambda P_0$$

$$\frac{dP_1}{dt} = 4\lambda P_0 - 3\lambda(1-c)P_F - 3\lambda c P_2$$

$$\frac{dP_2}{dt} = 3\lambda c P_1 - 2\lambda P_2$$

$$\frac{dP_F}{dt} = 3\lambda(1-c)P_1 + 2\lambda P_2(t)$$

- TAKE LAPLACE TRANSFORM AND MATRIX

  $s\underline{I} - T$

- TAKE INVERSE AND SOLVE.

$$P_0(t) = e^{-4\lambda t}$$

$$P_1(t) = 4e^{-3\lambda t}(1-e^{-\lambda t})$$

$$P_2(t) = 6c\,e^{-2\lambda t}(1-e^{-\lambda t})^2$$

$P_F(t) = \ldots\ldots$ etc.

REALISTIC MODEL OF COVERAGE

- SUPPOSE THAT REASON FOR COVERAGE IS THAT " τ " SECONDS NEEDED FOR FAILURE DETECTION.

- WHAT IF 2^{nd} FAILURE HAPPENS BEFORE FAULT IS DETECTED?

- $$C(t) = 0 \quad \text{if } t < \tau$$

$$= 1 - \frac{(1-e^{-\lambda\tau}) - P_s^2(t)(e^{-\lambda\tau}-1)}{[1-P(t)]^2}$$

for $t \geq \tau$

↪ TIME DEPENDENT TRANSITION RATES.