LECTURE #2

INTERLUDE:

MODELING AND EVALUATION OF RELIABLE SYSTEMS

READING:

e.g. K.S. TRIVEDI
PROBABILITY & STATISTICS WITH ~~[illegible]~~ RELIABILITY, QUEUEING, AND COMPUTER SCIENCE APPLICATIONS.
PRENTICE-HALL 1982

e.g. PERFORMANCE RELATED RELIABILITY MEASURES FOR COMPUTING SYSTEMS
M.D. BEAUDRY
IEEE TRANS. ON COMPUTERS
VC-27 NO 6, JUNE 1978.

INTERLUDE:

MODELING RELIABILITY IN DISTRIBUTED SYSTEMS: THE BASICS

ASSUMPTION: POISSON DISTRIBUTION OF FAILURES

- PROB OF FAILURE DURING INTERVAL Δt is APPROXIMATELY $z(t)\,\Delta t$

  $z(t)$ = "HAZARD" FUNCTION
- PROBABILITY OF TWO OR MORE FAILURES DURING Δt is NEGLIGIBLE.
- FAILURES ARE INDEPENDENT.

- DEFINE: $$m(t) = \int_0^t z(t)\,dt$$

- PROB. OF 'K' FAILURES IN INTERVAL $[0, t]$ is $$\frac{e^{-m(t)}[m(t)]^K}{K!}$$

- EXPECTED VALUE $$E[K] = \sum_{K=0}^{\infty} K \frac{e^{-m(t)}[m(t)]^K}{K!} = m(t)$$

- VARIANCE $$VAR[K] = E[K^2] - \left[E[K]\right]^2 = m(t)$$

- RELIABILITY $R(t) = $ PROB [0 FAILURES IN $[0,t]$] $$= e^{-m(t)}$$

SUBSTITUTE $K = 0$.

SPECIAL CASES:

CASE 1: HAZARD FUNCTION $z(t)$ IS CONSTANT.

$$z(t) = \lambda$$

CONSTANT FAILURE RATE

$\hookrightarrow$ $m(t) = \lambda t$

$\hookrightarrow$ "EXPONENTIAL"

$$PR\,[\,K \text{ FAILURES IN } (0,t)\,] = \frac{e^{-\lambda t}(\lambda t)^{K}}{K!}$$

$$E[K] = VAR[K] = \lambda t$$

$$\underline{R(t) = e^{-\lambda t}}$$

OTHER CASE: $z(t) = \alpha \lambda (\lambda t)^{\alpha - 1}$

"COMPONENT WEAROUT"

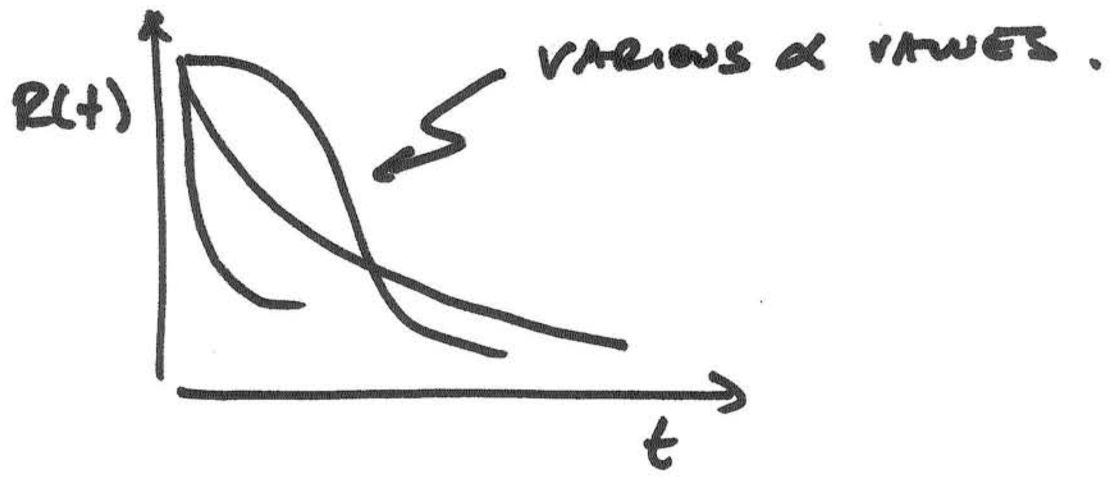
$\hookrightarrow \quad m(t) = (\lambda t)^{\alpha}$

$\hookrightarrow$ "WEIBULL"

$$PR[\text{K FAILURES}] = \frac{e^{-(\lambda t)^{\alpha}} (\lambda t)}{K!}$$

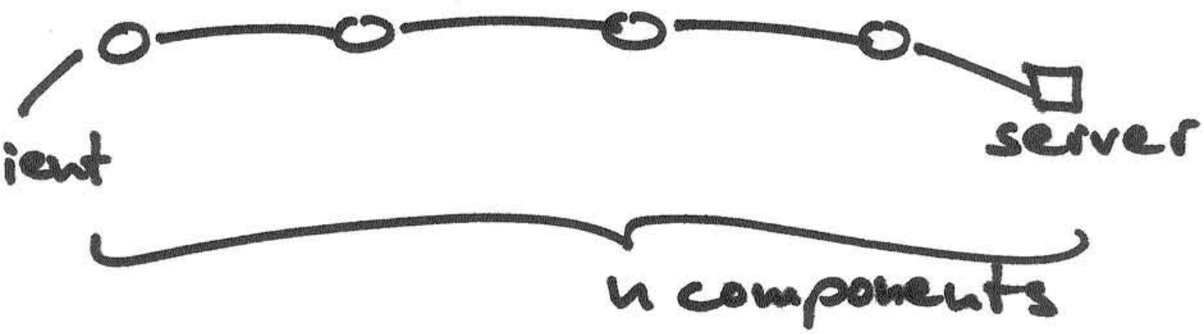
$$E[K] = VAR[K] = (\lambda t)^{\alpha}$$

$$R(t) = e^{-(\lambda t)^{\alpha}}$$



RELIABILITY OF NON-REDUNDANT SYSTEM

- SYSTEM HAS "n" COMPONENTS



- ALL COMPONENTS NEEDED TO "SURVIVE".

- $R_{SYSTEM}(t) = \prod_i R_i(t)$

- EXPONENTIAL : $\prod_i e^{-\lambda_i t} = e^{-(\Sigma \lambda_i)t}$

$\Rightarrow$ EFFECT IS <u>SUMMATION</u> OF FAILURE RATES OF INDIVIDUAL COMPONENTS !

SIMPLE MODELS

MTTF: MEAN TIME TO FAILURE.

$$MTTF = \int_0^\infty R(t)\,dt$$

"EXPONENTIAL"

$$= \int_0^\infty e^{-(\Sigma \lambda_i)t}\,dt$$

$$= \frac{1}{\sum_i \lambda_i}$$

COMBINATORIAL MODELING

ASSUMPTIONS

- SYSTEM PARTITIONED INTO "MODULES"
- MODULES FAIL INDEPENDENTLY
- ONCE MODEL FAILS, YIELDS INCORRECT RESULTS (NO BYZANTINE FAILURES).
- SUBSEQUENT FAILURES CANNOT BRING SYSTEM TO FUNCTIONAL STATE.

EXAMPLES:

- SERIES-PARALLEL SYSTEMS
- M-OUT-OF-N SYSTEMS
- ....

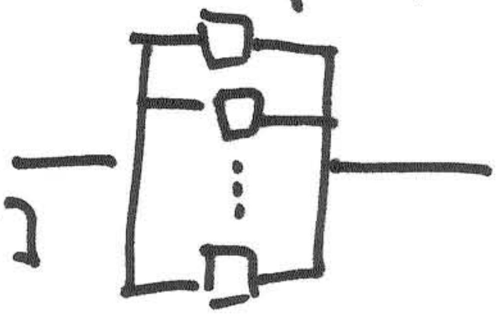
SERIES - PARALLEL SYSTEMS

- SERIES SYSTEM

$R_1(t)$ $\quad R_i(t)$ $\quad R_n(t)$

$$R_{SERIES}(t) = R_1(t) \times \dots R_i(t) \times \dots \times R_n(t) = \prod_i R_i(t)$$

- PARALLEL SYSTEM

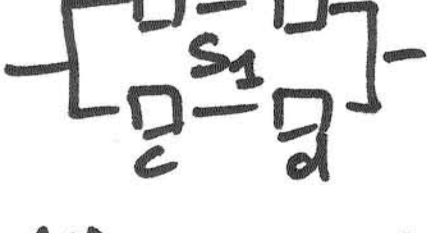


P[AT LEAST ONE NODE]

= 1 - P[none]

$$\hookrightarrow R_{PARALLEL} = 1 - \prod_i (1 - R_i(t))$$

- COMBINATIONS



vs.

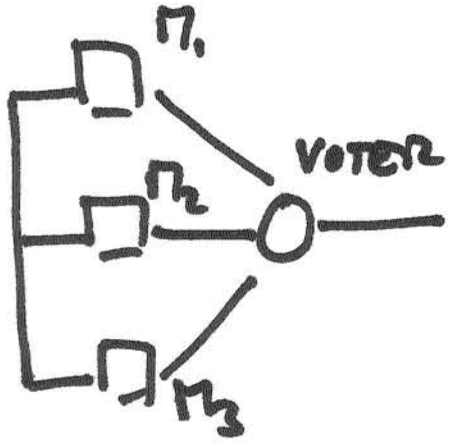
$$R_{S1}(t) = 1 - (1 - R_a R_b)(1 - R_c R_d)$$

$$R_{S2}(t) = (1 - (1 - R_a)(1 - R_c)) \times (1 - (1 - R_b)(1 - R_d))$$

M-OUT-OF-N SYSTEMS

OUT OF "M" WE NEED ONLY "N" TO FUNCTION CORRECTLY. e.g. TMR

$$R_{TMR} = \underbrace{R_m^3}_{\text{ALL 3 OPERATE}} + \underbrace{\binom{3}{1} R_m^2 (1 - R_m)}_{\text{ANY TWO OPERATE.}}$$



M-OUT-OF-N (CONT)

COMPARE SIMPLEX AND TMR :

$$R_{SIMPLEX}(t) = e^{-\lambda t}$$

$$MTTF_{SIMPLEX} = \int_0^{\infty} e^{-\lambda t}\, dt = \frac{1}{\lambda}$$

$$R_{TMR} = \left(e^{-\lambda t}\right)^3 + \binom{3}{1}\left(e^{-\lambda t}\right)^2\left(1 - e^{-\lambda t}\right)$$

$$= 3e^{-2\lambda t} - 2e^{-3\lambda t}$$

$$MTTF_{TMR} = \frac{3}{2\lambda} - \frac{2}{3\lambda} = \frac{5}{6\lambda}$$

?!

100% PERFECT VOTER

ILLUSTRATION: