**Problem Set 2**

CPSC 440/640 Quantum Algorithms

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**The assignment is due Monday, October 6, before class.**

1. Use the algorithm given give in the lecture notes "Controlled Unitary Gates" to derive an implementation of a controlled-Hadamard gate (controlled H gate) using single qubit and controlled-not gates.

2. For each computational basis state, trace the evolution of the state and prove that your implementation indeed provides an implementation of the controlled-Hadamard gate.

3. Can you find an implementation of the controlled-Hadamard gate with fewer controlled-not and single qubit gates? [Hint: Diagonalize $H$. Google eigenvectors and eigenvalues to find out how that is done.]

4. Let $x, y$ be vectors in $\mathbf{F}_2^n$, and let $s = x \oplus y$. Show that

$$H^{\otimes n}\left(\frac{1}{\sqrt{2}}|x\rangle + \frac{1}{\sqrt{2}}|y\rangle\right) = \frac{1}{\sqrt{2^{n-1}}}\sum_{z\in s^\perp}(-1)^{x\cdot z}|z\rangle.$$