Problem Set 4

CSCE 658 Randomized Algorithms

Due dates: Electronic submission of the .pdf file of this homework is due on **2/21/2019 before 2:00pm** on e-campus (as a turnitin assignment), a signed paper copy of the pdf file is due on **2/21/2019** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: _____

Read Chapter 4 in our textbook. If time permits, skim the first few chapters in a graduate textbook on probability theory.

Problem 1. A fixed point of a permutation $\pi\colon \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$ is a value x such that $\pi(x) = x$. Find (a) the expected number of fixed points and (b) the variance in the number of fixed points when the permutation is chosen uniformly at random from all permutations on n points.

Solution.

Problem 2. Show that one can efficiently simlulate choosing a random number from 1 to N using coin tosses. Specifically, show that for every integer $N \geq 2$ and $\delta > 0$ there is a randomized algorithm A running in $\mathrm{poly}(\log N \log(1/\delta))$-time with output in

$$\{1, 2, \ldots, N, ?\}$$

such that

(a) conditioned on not outputting ?, the output of A is uniformly distributed in $\{1, 2, \ldots, N\}$.

(b) the probability that A outputs ? is at most δ.

Solution.

Problem 3. Gain mastery (better reach 98 or more) in the probability theory topics on alcumus.

Checklist:
- ☐ Did you add your name?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted)
- ☐ Did you sign that you followed the Aggie honor code?
- ☐ Did you solve all problems?
- ☐ Did you submit the pdf file (resulting from your latex file) of your homework?
- ☐ Did you submit a hardcopy of the pdf file in class?