Problem Set 3

CSCE 658 Randomized Algorithms

Due dates: Electronic submission of the .pdf file of this homework is due on **2/14/2019 before 2:00pm** on e-campus (as a turnitin assignment), a signed paper copy of the pdf file is due on **2/14/2019** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: _____

Read Chapters 2 and 3 in our textbook. If time permits, skim the first few chapters in a graduate textbook on probability theory.

Problem 1. Master the 10 areas of probability theory in alcumus. The topics are given in ecampus. Compete for the first place in terms of accuracy and number of problems solved. [This part will go beyond the given deadline.]

Problem 2. Suppose that we independently roll two standard six-sided dice. Let X_1 be the number shown on the first die, X_2 the number shown on the second die, and $X = X_1 + X_2$.
(a) What is $\mathrm{E}[X \mid X_1 = X_2]$?
(b) What is $\mathrm{E}[X_1 \mid X = 9]$?

Solution.

Problem 3. Let $[a_1, a_2, \ldots, a_n]$ be an array of n distinct numbers. We say that a_i and a_j are inverted if $i < j$ but $a_i > a_j$. The *bubble sort* algorithm swaps pairwise adjacent inverted numbers in the list until there are no more inversions (that is, until the array is sorted). Suppose that the input is a random permutation, equally likely to be any of the $n!$ permutations of the n distinct numbers.
(a) Determine the expected number of inversions that need to be corrected by bubble sort.
(b) Determine the variance of the number of inversions that need to be corrected by bubble sort.

Solution.

Problem 4. Show that for a random variable X with standard deviation $\sigma[X]$ and any postive real number t that

$$\Pr[X - \mathrm{E}[X] \geq t\sigma[X]] \leq \frac{1}{1+t^2}.$$

Solution.

Homeworks must be typeset in LaTeX.
Checklist:
☐ Did you add your name?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit the pdf file (resulting from your latex file) of your homework?
☐ Did you submit a hardcopy of the pdf file in class?