Problem Set 1

CSCE 658 Randomized Algorithms

Due dates: Electronic submission of the .pdf file of this homework is due on **1/24/2019 before 2:00pm** on e-campus (as a turnitin assignment), a signed paper copy of the pdf file is due on **1/24/2019** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: _____

Get familiar with LaTeX. Read Chapter 1 and Section 2.1 in our textbook. If time permits, skim the first two chapters in a graduate textbook on probability theory. For example, [A. Gut, *Probability: A Graduate Course*, Springer, 2005] is available for free on libcat.

Problem 1. The number 1729 satiesfies $a^{1729} \equiv a \pmod{1729}$ for all integers, so it is a probable prime by Fermat's primality test. Find positive integers k and q such that $1729 - 1 = 2^k q$. Find the smallest integer $a \geq 2$ that is a Miller-Rabin witness to compositionality of 1729. Explain why the number a that you have found is a Miller-Rabin witness.

Problem 2. The Borel σ-algebra $\mathcal{B}(\mathbf{R})$ is the smallest σ-algebra on the set of real numbers that contains all open sets. Show that any countable subset of $\mathbf{R}$ is contained in the Borel σ-algebra $\mathcal{B}(\mathbf{R})$.

Solution.

Problem 3. How many equations does one have to check to establish that the n events $A_1, A_2, \ldots, A_n$ are independent?

Solution.

Problem 4. Prove the general inclusion-exclusion principle (Lemma 1.3 in our textbook) by induction.

Solution.

Problem 5. Suppose that the events $E_1, E_2, \ldots, E_n$ are independent. Show that the inclusion-exclusion formula from Lemma 1.3 in our textbook can be written in the much nicer form:

$$\Pr\left[\bigcup_{k=1}^{n} E_k\right] = 1 - \prod_{k=1}^{n}(1 - \Pr[E_k])).$$

Solution.

Homeworks must be typeset in LaTeX.

Checklist:

- ☐ Did you add your name?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted)
- ☐ Did you sign that you followed the Aggie honor code?
- ☐ Did you solve all problems?
- ☐ Did you submit the pdf file (resulting from your latex file) of your homework?
- ☐ Did you submit a hardcopy of the pdf file in class?