Problem Set 6

CSCE 658 Randomized Algorithms

Due dates: Electronic submission of the .pdf file of this homework is due on **3/8/2018 before 11:00am** on e-campus (as a turnitin assignment), a signed paper copy of the pdf file is due on **3/8/2018** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____________________________

Read Chapter 5 in our textbook.

Problem 1. Let X be a discrete random variable with the Poison distribution with parameter λ, that is,

$$\Pr[X = k] = e^{-\lambda}\frac{\lambda^k}{k!}.$$

Find the conditional expectation of X given that X is an even number.

Solution.

Problem 2. Let X be the sum of n independent $\{0,1\}$-valued random variables with $\mu = \mathrm{E}[X]$. One can deduce from the Chernoff bound

$$\Pr[X \geq (1+\delta)\mu] \leq \left(\frac{e^\delta}{(1+\delta)^{1+\delta}}\right)^\mu$$

the convenient version: For all real numbers $r \geq 2e\mu$, we have

$$\Pr[X \geq r] \leq 2^{-r}.$$

Find your own proof of this bound.

Solution.

Problem 3. A budget data center company offers a scalable dynamic load balancing system that supplies a new machine whenever a new job shows up, so that there are always n machines available to run n jobs. Unfortunately, one of the ways the company cuts cost is by not hiring programmers to replace the code from their previous randomized load balancing mechanism, so when the i-th job arrives, it is still assigned uniformly at random to one of the i available machines. This means that job 1 is always assigned to machine 1; job 2 is assigned with equal probability to machine 1 or 2; job 3 is assigned with equal probability to machine 1, 2, or 3; and so on. These choices are all independent.

If there are are n jobs, what is the expected load of the i-th machine? [Hint: Use an indicator random variable X_{ij} for the event that machine i gets job j.]

Solution.

Problem 4 (Continued)**.** The company claims that the maximum load is still not too bad. Justify this claim by showing that, with high probability, the most loaded machine after n jobs has arrived has load $O(\log n)$. Use the version of the Chernoff bound from Problem 2.

Solution.

Homeworks must be typeset in $\LaTeX$.

Checklist:

☐ Did you add your name?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit the pdf file (resulting from your latex file) of your homework?
☐ Did you submit a hardcopy of the pdf file in class?