Problem Set 4

CSCE 658 Randomized Algorithms

Due dates: Electronic submission of the .pdf file of this homework is due on **2/22/2018 before 11:00am** on e-campus (as a turnitin assignment), a signed paper copy of the pdf file is due on **2/22/2017** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: _____

Read Chapter 4 in our textbook. If time permits, skim the first few chapters in a graduate textbook on probability theory.

Problem 1. A fixed point of a permutation $\pi\colon \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$ is a value x such that $\pi(x) = x$. Find (a) the expected number of fixed points and (b) the variance in the number of fixed points when the permutation is chosen uniformly at random from all permutations on n points.

Solution.

Problem 2. Show that one can efficiently simlulate choosing a random number from 1 to N using coin tosses. Specifically, show that for every integer $N \geq 2$ and $\delta > 0$ there is a randomized algorithm A running in $\text{poly}(\log N \log(1/\delta))$-time with output in

$$\{1, 2, \ldots, N, ?\}$$

such that

(a) conditioned on not outputting ?, the output of A is uniformly distributed in $\{1, 2, \ldots, N\}$.

(b) the probability that A outputs ? is at most δ.

Solution.

Problem 3. Gain mastery (better reach 98 or more) in "Basic Probability as Counting" on alcumus.

Problem 4. Gain mastery (better reach 98 or more) in "Basic Probability with Combinations" on alcumus.

Problem 5. Gain mastery (better reach 98 or more) in "Probability with Casework " on alcumus.

Problem 6. Gain mastery (better reach 98 or more) in "Complementary Probability" on alcumus.

Problem 7. Gain mastery (better reach 98 or more) in "Expected Value" on alcumus.

Feel free to work on the counting problems as well. You compete against your classmates. I will regularly post the top 3.

Homeworks must be typeset in LaTeX.

Checklist:

☐ Did you add your name?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit the pdf file (resulting from your latex file) of your homework?
☐ Did you submit a hardcopy of the pdf file in class?