Problem Set 2

CSCE 658 Randomized Algorithms

Due dates: Electronic submission of the .pdf file of this homework is due on **2/8/2018 before 11:00am** on e-campus (as a turnitin assignment), a signed paper copy of the pdf file is due on **2/8/2017** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: _____

Read Chapter 2 in our textbook. If time permits, skim the first few chapters in a graduate textbook on probability theory.

Problem 1. Let $G = (V, E)$ be a connected multigraph with $n = |V|$ vertices. A cut in G is a partition of the set of vertices V into two nonempty parts V_1 and V_2. In other words, we have

$$V = V_1 \cup V_2, \quad V_1 \cap V_2 = \emptyset,$$

as well as $V_1 \neq \emptyset$ and $V_2 \neq \emptyset$. Count the total number of cuts in G. [Hint: It is an exponential number of cuts, but you should give a very precise count here.]

Solution.

Problem 2. Let us keep the notation of the previous problem. Use the knowledge that you have gained from the lecture to prove that there are at most a polynomial number of minimum cuts in G. Actually, give a more precise upper bound.

Solution.

Problem 3. The FastCut algorithm by Karger and Stein finds a given minimum cut C with probability $P(n) \geq c/\ln n$ for some positive real number c.

(a) How many times should you repeat FastCut so that the probability to miss a given minimum cut is less than $1/n^3$? [Hint: Put the formula $1 + x \leq e^x$ to good use].

(b) Using the number of repetitions in part (a) to determine the probability to miss **any** minimum cut in G.

You should be able to conclude that you can find all minimum cuts with high probability (i.e., with probability $\to 1$ as $n \to \infty$) when FastCut is repeated the number of times given in part (a).

Problem 4. Suppose that Alice wants to send Bob a bit through a network of n relays. A recent update of the firmware left each relay somewhat unreliable. Now a relay will flip the bit with probability p. What is the probability that Bob will receive the correct bit? Prove your result.

Solution.

Problem 5. Suppose that we roll a fair k-sided die with the numbers 1 through k on the die's faces. If X is the number that appears, what is $\mathrm{E}[X]$?

Solution.

Homeworks must be typeset in LaTeX.

Checklist:

- ☐ Did you add your name?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted)
- ☐ Did you sign that you followed the Aggie honor code?
- ☐ Did you solve all problems?
- ☐ Did you submit the pdf file (resulting from your latex file) of your homework?
- ☐ Did you submit a hardcopy of the pdf file in class?