

Problem Set 1

CSCE 658 Randomized Algorithms

Due dates: Electronic submission of the .pdf file of this homework is due on **2/1/2018 before 11:00am** on e-campus (as a turnitin assignment), a signed paper copy of the pdf file is due on **2/1/2017** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: _____

Get familiar with \LaTeX . Read Chapter 1 and Section 2.1 in our textbook. If time permits, skim the first two chapters in a graduate textbook on probability theory. For example, [A. Gut, *Probability: A Graduate Course*, Springer, 2005] is available for free on libcat.

Problem 1. The Borel σ -algebra $\mathcal{B}(\mathbf{R})$ is the smallest σ -algebra on the set of real numbers that contains all open sets. Show that any countable subset of \mathbf{R} is contained in the Borel σ -algebra $\mathcal{B}(\mathbf{R})$.

Solution.

Problem 2. How many equations does one have to check to establish that the n events A_1, A_2, \dots, A_n are independent?

Solution.

Problem 3. Prove the general inclusion-exclusion principle (Lemma 1.3 in our textbook) by induction.

Solution.

Problem 4. Suppose that the events E_1, E_2, \dots, E_n are independent. Show that the inclusion-exclusion formula from Lemma 1.3 in our textbook can be written in the much nicer form:

$$\Pr \left[\bigcup_{k=1}^n E_k \right] = 1 - \prod_{k=1}^n (1 - \Pr[E_k]).$$

Solution.

Problem 5. Let (Ω, \mathcal{F}) be a measurable space. Suppose that $X, Y : \Omega \rightarrow \mathbf{R}$ are random variables. Show that $\max\{X, Y\}$ and $\min\{X, Y\}$ are random variables. [Hint: You need to establish that $\max\{X, Y\} \leq z$ and $\min\{X, Y\} \leq z$ define events in \mathcal{F} .]

Solution.

Homeworks must be typeset in \LaTeX .

Checklist:

- Did you add your name?
- Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
- Did you sign that you followed the Aggie honor code?
- Did you solve all problems?
- Did you submit the pdf file (resulting from your latex file) of your homework?
- Did you submit a hardcopy of the pdf file in class?