Teleportation

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The Problem

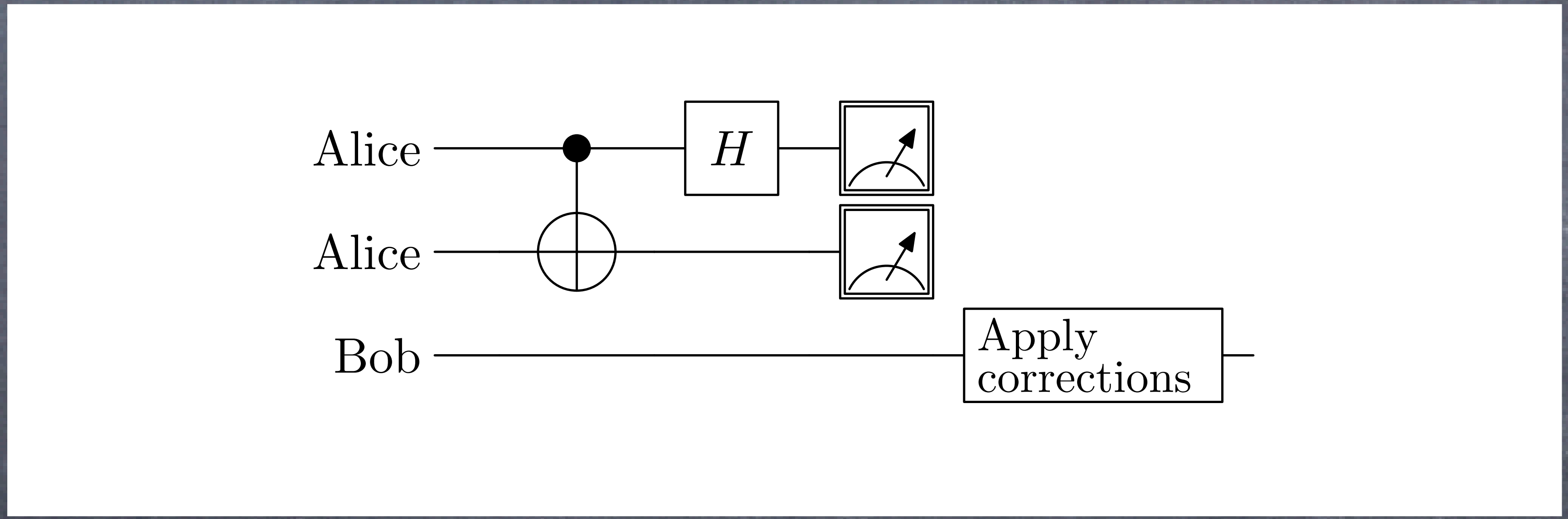
Alice wants to send the state of a quantum bit to Bob.

We assume that they share a pair of entangled quantum bits in the state $(|00\rangle+|11\rangle)/\sqrt{2}$.

How can they do it if classical communication is allowed?

The Quantum Circuit

Let's assume that Alice wants to teleport a quantum bit in the state $a|0\rangle + b|1\rangle$ to Bob and that they share a pair of entangled quantum bits such that the system is in the state: $(a|0\rangle+b|1\rangle) \otimes (|00\rangle+|11\rangle)/\sqrt{2}$. We claim that the following quantum circuit can solve the problem:



State Evolution (1)

Initial state:

$$(a|0\rangle + b|1\rangle) \otimes (\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle).$$

Applying the controlled not yields

$$a|0\rangle \otimes (\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle) \ + \ b|1\rangle \otimes (\frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|01\rangle).$$

State Evolution (2)

$$a|0\rangle \otimes (\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle) \;+\; b|1\rangle \otimes (\frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|01\rangle).$$

Applying the Hadamard gate on the most significant qubit yields the state

$$\begin{aligned} a(\tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{1}{\sqrt{2}}|1\rangle) \otimes (\tfrac{1}{\sqrt{2}}|00\rangle + \tfrac{1}{\sqrt{2}}|11\rangle) \\ + b(\tfrac{1}{\sqrt{2}}|0\rangle - \tfrac{1}{\sqrt{2}}|1\rangle) \otimes (\tfrac{1}{\sqrt{2}}|10\rangle + \tfrac{1}{\sqrt{2}}|01\rangle). \end{aligned}$$

Rewriting the State

Rewriting the state yields

$$a(\tfrac{1}{2}|000\rangle + \tfrac{1}{2}|011\rangle + \tfrac{1}{2}|100\rangle + \tfrac{1}{2}|111\rangle)$$
$$+b(\tfrac{1}{2}|001\rangle + \tfrac{1}{2}|010\rangle - \tfrac{1}{2}|101\rangle - \tfrac{1}{2}|110\rangle).$$

or

$$\frac{1}{2}\Big(|00\rangle \otimes (a|0\rangle + b|1\rangle) + |01\rangle \otimes (a|1\rangle + b|0\rangle)$$
$$+|10\rangle \otimes (a|0\rangle - b|1\rangle) + |11\rangle \otimes (a|1\rangle - b|0\rangle)\Big).$$

Measurement and Correction

$$\frac{1}{2}\Big(|00\rangle \otimes (a|0\rangle + b|1\rangle) + |01\rangle \otimes (a|1\rangle + b|0\rangle)$$
$$+|10\rangle \otimes (a|0\rangle - b|1\rangle) + |11\rangle \otimes (a|1\rangle - b|0\rangle)\Big).$$

measuring the two most significant quantum bits yields

Observation	Resulting State	Alice tells Bob
00	$\lvert 00\rangle \otimes (a\lvert 0\rangle + b\lvert 1\rangle)$	to do nothing
01	$\lvert 01\rangle \otimes (a\lvert 1\rangle + b\lvert 0\rangle)$	to apply X
10	$\lvert 10\rangle \otimes (a\lvert 0\rangle - b\lvert 1\rangle)$	to apply Z
11	$\lvert 11\rangle \otimes (a\lvert 1\rangle - b\lvert 0\rangle)$	to apply ZX