# Quantum Bits

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# Quantum Bit

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A quantum bit has two clearly distinguishable states, $|0\rangle$ and $|1\rangle$. In addition it can be in any state $a|0\rangle + b|1\rangle$ with $|a|^2 + |b|^2 = 1$.

The observed value of a quantum bit is always 0 or 1, never anything else.

# Quantum Bits

The states $|0\rangle$ and $|1\rangle$ should be understood as basis vectors of a complex vector space. By convention, we order the basis $(|0\rangle, |1\rangle)$ such that

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

The state $a|0\rangle + b|1\rangle$ is a linear combination of these vectors and is represented as

$$a|0\rangle + b|1\rangle = \begin{pmatrix} a \\ b \end{pmatrix}.$$

# Measurement

We call { $|0\rangle$, $|1\rangle$ } the computational basis of a quantum bit.

A measurement of a quantum bit in the state $a|0\rangle + b|1\rangle$ will give

- the value 0 with probability $|a|^2$ and

- the value 1 with probability $|b|^2$.

# Perfect Coin Flip

On a classical computer, we usually do not have a good source of random bits available, and we simulate randomness by pseudo-random generators.

On a quantum computer, we can get a random generator by initializing and measuring a quantum bit in the state:

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

# Exercise

Find <u>all</u> quantum states that yield

- 0 with probability 1/2 and

- 1 with probability 1/2.