Problem Set 9

Due dates: Electronic submission of this homework is due on **Friday 11/10/2017 before 2:50pm** on ecampus, a signed paper copy of the pdf file is due on **11/10/2017** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____________________________

Problem 1 (10 points)**.** Solve Exercise 2.7 of the randomized algorithms lecture notes (on perusall.com).

Solution.

Problem 2 (10 points)**.** Solve Exercise 2.8 of the randomized algorithms lecture notes (on perusall.com).

Solution.

Problem 3 (20 points)**.** Solve Exercise 2.9 of the randomized algorithms lecture notes (on perusall.com).

Solution.

Problem 4 (10 points)**.** Suppose that the running time of a randomized algorithm is modeled by the random variable X. You have determined the expected running time $\mathrm{E}[X]$. Use Markov's inequality to bound the probability that an execution of the randomized algorithm equals or exceeds $(1+\epsilon)\mathrm{E}[X]$.

Solution.

Problem 5 (10 points)**.** Suppose that the running time of a randomized algorithm is modeled by the random variable X. Suppose that you know the expected running time $\mathrm{E}[X]$. You know that your algorithm has a worst case running time that exceeds $\mathrm{E}[X]$ by a significant margin. Therefore, you decide to stop the execution whenever it exceeds $(1+\epsilon)\mathrm{E}[X]$ steps, and restart the algorithm. You will repeat the execution of the algorithm at most t times. Denote by X_k the random variable modeling running time of the k-th try. Determine the probability that the randomized algorithm equals or exceeds $(1+\epsilon)\mathrm{E}[X]$ steps in all t repetitions. This probability models the failure probability of this algorithm.

Solution.

Problem 6 (20 points)**.** How should we choose the number of repetitions t such that the randomized algorithm has with probability $1 - 1/n$ at least one run such that $X < (1 + \epsilon)\mathrm{E}[X]$ among the t runs.

Solution.

Problem 7 (20 points)**.** How many people need to be in a room such that two of those people share a birthday with probability of 97%. We assume that birthdays are uniformly distributed. You need to derive your result.

Solution.

Checklist:
- ☐ Did you add your name?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted)
- ☐ Did you sign that you followed the Aggie honor code?
- ☐ Did you solve all problems?
- ☐ Did you submit the pdf file of your homework?
- ☐ Did you submit a hardcopy of the pdf file in class?