Problem Set 7

Due dates: Electronic submission of this homework is due on **Tuesday 4/2/2019 before 12:30pm** on ecampus, a signed paper copy of the pdf file is due on **4/2/2019** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____________________________

Problem 1 (20 points)**.** Suppose that the sample space Ω is given by the set of positive integers. Let $\mathcal{F}$ denote the smallest family of subsets of Ω such that (a) $\mathcal{F}$ contains all finite sets, (b) $\mathcal{F}$ is closed under complements (meaning if A is in $\mathcal{F}$, then A^c is in $\mathcal{F}$), and (c) $\mathcal{F}$ is closed under countable unions (so if the sets $E_1, E_2, \ldots$ are contained in $\mathcal{F}$, then $\bigcup_{k=1}^{\infty} E_k$ is contained in $\mathcal{F}$).
(a) Show that $\mathcal{F}$ is a σ-algebra.
(b) Prove of disprove: $\mathcal{F}$ is equal to the power set $P(\Omega)$.

Solution.

Problem 2 (20 points)**.** Suppose that A and B are events in an experiment with $\Pr[A \setminus B] = 1/6$, $\Pr[B \setminus A] = 1/4$, and $\Pr[A \cap B] = 1/12$. Find the probability of each of the following events:
(a) A,
(b) B,
(c) $A \cup B$,
(d) $A^c \cup B^c$.

Solution.

Problem 3 (20 points)**.** Give examples of events where (a) $\Pr[A \mid B] < \Pr[A]$, (b) $\Pr[A \mid B] = \Pr[A]$, and (c) $\Pr[A \mid B] > \Pr[A]$. Make sure that your proofs are complete and self-contained.

Solution.

Problem 4 (20 points)**.** There may be several different min-cut sets in a graph. Using the analysis of the randomized min-cut algorithm, argue that there can be at most $n(n-1)/2$ distinct min-cut sets.

Solution.

Problem 5 (20 points)**.** The FastCut algorithm by Karger and Stein finds a given minimum cut C with probability $P(n) \geq c/\ln n$ for some positive real number c. How many times should you repeat FastCut so that the probability to miss a given minimum cut is less than $1/n$? [Hint: Put the formula $1+x \leq e^x$ to good use].

Solution.

Checklist:
☐ Did you add your name?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit the pdf file of your homework?
☐ Did you submit a hardcopy of the pdf file in class?