Problem Set 3

Due dates: Electronic submission of the pdf file of this homework is due on **2/14/2019 before noon** on ecampus, a signed paper copy of the pdf file is due on **2/14/2019** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____________________________

Make sure that you describe all solutions in your own words. Read chapters 30 and 16 in our textbook.

Problem 1. (a) (10 points) Suppose that you are given a polynomial

$$A(x) = \sum_{k=0}^{n-1} a_k x^k.$$

The input to the FFT of length n is given by an array containing the coefficients $(a_0, \ldots, a_{n-1})$. Describe the output of the FFT in terms of the polynomial $A(x)$.

(b) (15 points) Let ω be a primitive nth root of unity. The fast Fourier transform implements the multiplication with the matrix

$$F = (\omega^{ij})_{i,j\in[0..n-1]}.$$

Show that the inverse of the F is given by

$$F^{-1} = \frac{1}{n}(\omega^{-jk})_{j,k\in[0..n-1]}$$

[Hint: $x^n - 1 = (x-1)(x^{n-1} + \cdots + x + 1)$, so any power $\omega^\ell \neq 1$ must be a root of $x^{n-1} + \cdots + x + 1$.] Thus, the inverse FFT, called IFFT, is nothing but the FFT using ω^{-1} instead of ω, and multiplying the result with $1/n$.

(c) (10 points) Describe how to do a polynomial multiplication using the FFT and IFFT for polynomials $A(x)$ and $B(x)$ of degree $\leq n-1$. Make sure that you describe the length of the FFT and IFFT needed for this task.

(d) (15 points) How can you modify the polynomial multiplication algorithm based on FFT and IFFT to do multiplication of long integers in base 10? Make sure that you take care of carries in a proper way.

Solution.

Problem 2. (20 points) You overhear a conversation where someone mentions that Morgenstern proved an $\Omega(n \log n)$ lower bound on the fast Fourier transform and someone else mentions that a group of MIT researchers found in 2012 a faster than fast Fouier transform that is $o(n \log n)$. These two comments seem to contradict each other. Do your research and find out what Morgenstern really proved and under what circumstances the MIT algorithm can improve on the FFT.

Problem 3. Solve the following two problems on matroids.

1. (10 points) Solve exercise 16.4-3 on page 443 of [CRLS].
2. (20 points) Solve exercise 16.4-4 on page 443 of [CLRS].

Solution.

Discussions on ecampus are always encouraged, especially to clarify concepts that were introduced in the lecture. However, discussions of homework problems on ecampus should not contain spoilers. It is okay to ask for clarifications concerning homework questions if needed.

Checklist:
☐ Did you add your name?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit (a) the pdf file of your homework?
☐ Did you submit (b) a hardcopy of the pdf file in class?