Problem Set 1

Due dates: Electronic submission of .tex and .pdf files of this homework is due on **1/24/2014 before 11:00am** on e-campus (as a turnitin assignment), a signed paper copy of the pdf file is due on **1/24/2014** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____________________________

Problem 1. (15 points) Get familiar with LaTeX. Nicely typeset the definition of e.

[Incidentally, we recommend that you keep a LaTeX file with all definitions and important theorems that we learn in this class. This will help you to memorize the definitions, and will allow you to quickly access this information when solving homework problems.]

Solution.

Problem 2. (15 points) In each of the following situations, decide whether $f = O(g)$, $f = \Omega(g)$, or both (in which case $f = \Theta(g)$).
(a) $f(n) = n^3$ and $g(n) = n^2 \log n$.
(b) $f(n) = n!$ and $g(n) = 2^n$.
(c) $f(n) = n^{1000}$ and $g(n) = 2^n$.
(d) $f(n) = \sqrt{n^2 + 1}$ and $g(n) = n/2$.
(e) $f(n) = \log_{10}(n)$ and $g(n) = \ln(n)$.
No proofs need to be given.

Solution.

Problem 3. (10 points) Use the definitions to prove the following facts.
(a) $H_n = O(\log n)$, where $H_n = 1 + 1/2 + 1/3 + \cdots 1/n$.
(b) $H_n = \Omega(\log n)$, where $H_n = 1 + 1/2 + 1/3 + \cdots 1/n$.
Hint: Compare with an integral!

Problem 4. (10 points) Use the lim, lim sup, or lim inf criteria to prove the following facts:
(a) $3n^2 + 5n + \log n = O(n^2)$.
(b) $n^2 + (1 + (-1)^n)n = O(n^2)$.

Problem 5. (20 points)
(a) Verify that

$$n^{\underline{3}} + 3n^{\underline{2}} + n^{\underline{1}} = n^3.$$

(b) Use this fact to find a formula for

$$\sum_{k=0}^{n} k^3$$

using the finite difference calculus.

Problem 6. (20 points) The polynomials of degree m in the variable n can be expressed as a linear combination of either $\{n^0, n^1, n^2, \ldots, n^m\}$ or $\{n^{\underline{0}}, n^{\underline{1}}, n^{\underline{2}}, \ldots, n^{\underline{m}}\}$. Therefore, there exist integers $\left\{ {m \atop k} \right\}$ such that

$$n^m = \sum_{k=0}^{m} \left\{ {m \atop k} \right\} n^{\underline{k}}.$$

Define

(a) $\left\{ {0 \atop 0} \right\} = 1$
(b) $\left\{ {m \atop 0} \right\} = 0$ if $m > 0$
(c) $\left\{ {m \atop k} \right\} = 0$ if $k < 0$ or $k > m$.
Show that

$$\left\{ {m \atop k} \right\} = k \left\{ {m-1 \atop k} \right\} + \left\{ {m-1 \atop k-1} \right\}$$

holds.

Problem 7. (10 points) Use the recursion from the previous problem to find a closed from for

$$\sum_{k=0}^{n} k^4$$

using the finite difference calculus.

I will allow that you explore some of the problems in class together with your team, **but** the homework solution must be formulated by yourself. Homeworks must be typeset in LaTeX.

Solution.

Checklist:
☐ Did you add your name?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit (a) your latex source file and (b) the resulting pdf file of your homework?
☐ Did you submit (c) a hardcopy of the pdf file in class?