One can show the following relation between uniform and nonuniform distribution case:

$$\begin{aligned} p_{uni} &= 1 - \frac{n(n-1)\cdots(n-m+1)}{n^m} \\ &= 1 - m! e_m(1/n, 1/n, \ldots, 1/n) \\ &\leq 1 - m! e_m(p_1, p_2, \ldots, p_n) = p_{nu}, \end{aligned}$$

as e_m is a so-called Schur-concave function.