

Problem Set 9

Due dates: Electronic submission of this homework is due on **Thursday 11/8/2017 before 11:00am** on ecampus, a signed paper copy of the pdf file is due on **11/8/2017** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: _____

Problem 1 (10 points). Solve Exercise 2.7 of the randomized algorithms lecture notes (on perusall.com).

Solution.

Problem 2 (10 points). Solve Exercise 2.8 of the randomized algorithms lecture notes (on perusall.com).

Solution.

Problem 3 (20 points). Solve Exercise 2.9 of the randomized algorithms lecture notes (on perusall.com).

Solution.

Problem 4 (10 points). Suppose that the running time of a randomized algorithm is modeled by the random variable X . You have determined the expected running time $E[X]$. Use Markov's inequality to bound the probability that an execution of the randomized algorithm **equals or** exceeds $(1 + \epsilon)E[X]$.

Solution.

Problem 5 (10 points). Suppose that the running time of a randomized algorithm is modeled by the random variable X . Suppose that you know the expected running time $E[X]$. You know that your algorithm has a worst case running time that exceeds $E[X]$ by a significant margin. Therefore, you decide to stop the execution whenever it exceeds $(1 + \epsilon)E[X]$ steps, and restart the algorithm. You will repeat the execution of the algorithm at most t times. Denote by X_k the random variable modeling running time of the k -th try. Determine the probability that the randomized algorithm **equals or** exceeds $(1 + \epsilon)E[X]$ steps in all t repetitions. This probability models the failure probability of this algorithm.

Solution.

Problem 6 (20 points). How should we choose the number of repetitions t such that the randomized algorithm has with probability $1 - 1/n$ at least one run such that $X < (1 + \epsilon)E[X]$ among the t runs.

Solution.

Problem 7 (20 points). How many people need to be in a room such that two of those people share a birthday with probability of 97%. We assume that birthdays are uniformly distributed. You need to derive your result.

Solution.

Checklist:

- Did you add your name?
- Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
- Did you sign that you followed the Aggie honor code?
- Did you solve all problems?
- Did you submit the pdf file of your homework?
- Did you submit a hardcopy of the pdf file in class?