Problem Set 2

Due dates: Electronic submission of .pdf files of this homework is due on **9/13/2018 before 11:00am** on ecampus, a signed paper copy of the pdf file is due on **9/13/2018** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____________________________

Problem 1 (15 points)**.** You need to solve the following problem: Is a given positive integer n equal to one of the primes in the set $P = \{p_1, p_2, \ldots, p_k\}$. You are only allowed to perform comparisons such as $n < p_j$ or $n = p_j$. Find the decision tree for binary search on a sorted array containing the elements of P in order, where

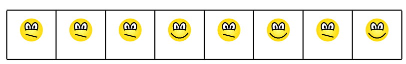
$$P = \{2, 3, 7, 11, 17, 23, 31, 37\}.$$

For your answer, you should draw this decision tree. You should keep in mind that binary search correctly identifies any positive integer n such that $n \notin P$.

[Hint: Use the cool tikz package to draw the decision tree in LaTeX.]

Problem 2 (15 points)**.** You need to solve the following problem: Is a given positive integer n equal to one of the primes in the set $P = \{p_1, p_2, \ldots, p_k\}$. You are only allowed to perform comparisons such as $n < p_j$ or $n = p_j$. Derive a tight lower bound on the number of comparisons need by any algorithm to solve this problem using a decision tree.

Problem 3 (10 points)**.** Amelia attempted to solve n algorithmic problems. She wrote down one problem per page in her journal and marked the page with  when she was unable to solve the problem and with  when she was able to solve it. So the pages of her journal look like this:



Use an adversary method to show that any method to find a page with an  smiley on it might have to look at all n pages.

Problem 4 (20 points)**.** Amelia attempted to solve n algorithmic problems, where n is an odd number. She wrote down one problem per page in her journal and marked the page with  when she was unable to solve the problem and with  when she was able to solve it. Suppose that we want to find the pattern , where she was unable to solve a problem, but was able to solve the subsequent problem.

Find an algorithm that always looks at fewer than n pages but is able to correctly find the pattern when it exists. [Hint: First look at all even pages.]

Problem 5. (20 points) Give a $(2n-1)$ lower bound on the number of comparisons needed to merge two sorted lists $(a_1, a_2, \ldots, a_n)$ and $(b_1, b_2, \ldots, b_n)$ with $a_1 < a_2 < \cdots < a_n$ and $b_1 < b_2 < \cdots < b_n$. [Hint: Use an adversarial method. Why can't you have in general $2n - 2$ or fewer comparisons? If you are stuck, merge (1,3,5,7) and (2,4,6,8) and see how an adversary could modify this input if less than 7 comparisons were made.]

Solution.

Problem 6. (20 points) Solve Exercise 8.1-4 on page 194 of our textbook.

Solution.

Checklist:

☐ Did you add your name?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you write the solution in your own words?
☐ Did you submit the pdf file of your homework?
☐ Did you submit a signed hardcopy of the pdf file in class?