Problem Set 3
CPSC 411 Analysis of Algorithms
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The assignment is due next Friday, Sep 23, 2011, before class.

Exercise 1 (10 points). *Find the smallest matroid containing the sets* $\{a, b\}$ *and* $\{b, c\}$.

Exercise 2 (20 points). *Let* a *and* b *be positive integers. Let* S *be a finite nonempty set,* A *and* B *disjoint subsets of* S *such that* $S = A \cup B$. *Let* $F = \{C \subseteq S \mid |C \cap A| \leq a \text{ and } |C \cap B| \leq b\}$. *Show that* (S, F) *is a matroid.*

Exercise 3 (10 points). *If the sets* A *and* B *in the previous exercise are not disjoint, can we still always conclude that* (S, F) *is a matroid?*

Exercise 4 (20 points). *Keep the notation of exercise 2. Let* $a = 2$ *and* $b = 2$. *Let* $A = \{1, 2, 3, 4\}$, $B = \{5, 6, 7, 8\}$, *and*$S = A \cup B$. *Let* $w \colon S \to \mathbf{R}$ *be the weight function* $w(x) = x$ *if* x *is even, and* $w(x) = 2x$ *if* x *is odd.*
(a) Explicitly write down the matroid.
(b) Write down all the steps of the algorithm Greedy((S,F),w)
(c) What is the result returned by the algorithm?

Exercise 5 (10 points). *Suppose you have a set of denomiations with values* $1, 5, 7$. *Does the Greedy algorithm to give change always return the least possible number of coins? Prove the result or give the smallest possible counter example (smallest amount* C*).*

Exercise 6 (10 points). *Multiply the matrices*

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 4 & 4 \end{pmatrix}$$

Do this by hand. Explain how one can obtain the entries of the resulting matrix.

Exercise 7 (20 points). *Suppose that you have a* 5×2 *matrix* A_1, *a* 2×10 *matrix* A_2, *a* 10×25 *matrix* A_3, *and a* 25×10 *matrix* A_4. *Determine the minimal number of scalar multiplications that are needed to form the product*

$$A_1 A_2 A_3 A_4$$

using the algorithm given in the lecture. Do this exercise by hand rather than on the computer. Show your work!