Problem Set 1
CPSC 411 Analysis of Algorithms
Andreas Klappenecker

The assignment is due next Friday, Sep 9, 2011, before class.

Exercise 1 (10 points). *Use the definition of Big-Oh notation to prove that*

$$n\lfloor logn \rfloor = O(n^2).$$

Here $\lfloor x \rfloor$ *denotes the floor function that yields the largest integer* $\leq x$ *as a value.*

Exercise 2 (10 points). *Prove that*

$$n\lfloor logn \rfloor = \Theta(n \log n).$$

Exercise 3 (10 points). *Prove or disprove:* $2^n \log n = \Omega(e^n)$.

Exercise 4 (20 points). *Show that for fixed* k*, we have*

$$\binom{n}{k} = \frac{n^k}{k!} + O(n^{k-1}) \quad \text{and} \quad \binom{n+k}{k} = \frac{n^k}{k!} + O(n^{k-1}),$$

where $\binom{n}{k} = n(n-1)(n-2)\cdots(n-k+1)/k!$ *is the binomial coefficient.*

Exercise 5 (20 points). *Suppose that* f *and* g *are function from the natural numbers to the positive real numbers. Suppose that the limit* $\lim_{n\to\infty} f(n)/g(n)$ *exists and is positive. Prove or disprove that* $O(f) = O(g)$.

Exercise 6 (10 points). *Suppose that it is known that each of the items in an array* $a[1..n]$ *has one of two distinct values. Give a sorting algorithm for such arrays that takes time proportional to* n.

Exercise 7 (20 points). *Assume that the running time of Mergesort is* $cn \log n + dn$*, where* c *and* d *are machine-dependent constants. Show that if we implement the program on a particular machine and observe a running time* t_n *for some value of* n*, then we can accurately estimate the running time for* $2n$ *by* $2t_n(1 + 1/\log n)$*, independent of the machine.*