Exam 2

CPSC 311 Analysis of Algorithms
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- This exam contains 1+6 problems. You have 75 minutes to earn 100 points.
- This exam is closed book.
- You are allow to use a nonprogrammable calculator, although you will not need one.
- Do not spend too much time on a single problem. Read them all through first and attack them in the order that allows you to make the most progress.
- Show your work, as partial credit will be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it. Be neat.
- Recall the Aggie code of honor. Cheating will have severe consequences.

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The work shown in this exam is my own.

Signature required: ________________________________

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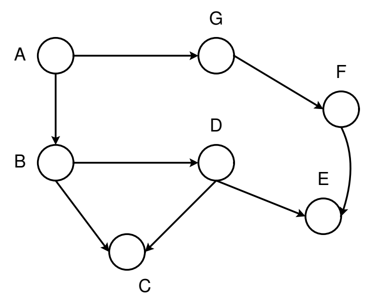
Problem 0 (2 points) Write your name and TAMU ID on each page.

Problem 1 (13 points) Consider the graph



a) Draw the result of topological sorting. All vertices should be lined up on the straight line below. Include edges of the original graph for clarity.

b) How is topological sorting of a directed graph $G = (V, E)$ implemented? Give a clear and brief description of the algorithm.

c) What is the time-complexity of topologically sorting a direct graph $G = (V, E)$ using the algorithm discussed in class? (Oh-notation, tight bound)

d) When is it possible to topologically sort a directed graph $G = (V, E)$?

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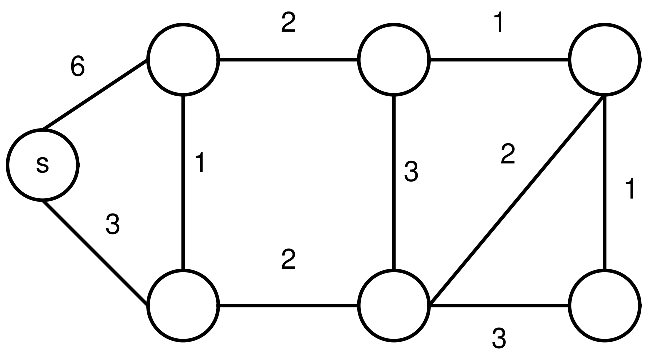
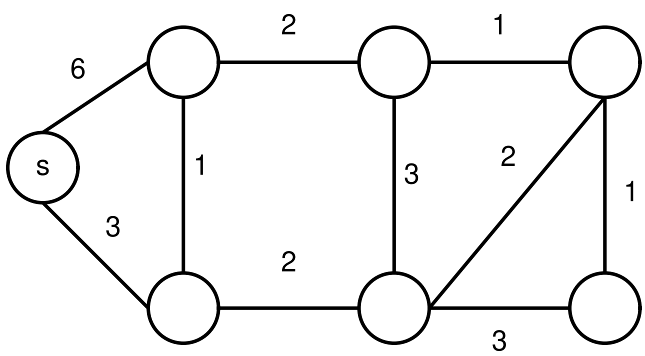
Problem 2 (15 points) Determine the minimum spanning tree of the following graph with the help of a) Prim's and b) Kruskal's algorithm. Clearly mark the edges of the resulting spanning tree. Label the edges in the order of selection, beginning with 1. Use the node labeled s in case the algorithm needs a starting node.

a) Prim's algorithm



b) Kruskal's algorithm



c) Briefly explain the difference between the two algorithms.

d) Which algorithm is asymptotically faster?

Problem 3 (25 points) A flow network is given by a directed graph $G = (V, E)$, which is equipped with two distinct vertices $s, t \in V$, called the source and the sink vertex, respectively, and by a capacity function $c : V \times V \to \mathbf{Z}$, that is, $c(u, v) \geq 0$ for all $u, v \in V$, and $c(u, v) = 0$ if $(u, v) \notin E$.

a) An admissible flow $f\colon V \times V \to \mathbf{Z}$ has to satisfy which three conditions?

b) How is the residual network G_f of a flow network $G = (V, E)$ with capacity c and admissible flow f defined?

i) Residual capacity: $c_f(u, v) =$

ii) Residual network: $G_f = (V, E_f)$, $\qquad E_f = \{(u, v) \mid \qquad\qquad \}$

c) Give a small example of a flow network, where the Ford-Fulkerson method might need exactly $2,000,000$ iterations, when the admissible paths are chosen in the wrong way.

d) What is the running time of the basic Ford-Fulkerson algorithm?

e) What is the maximal flow through the network given in part f). Determine a bound without using the Ford-Fulkerson algorithm.

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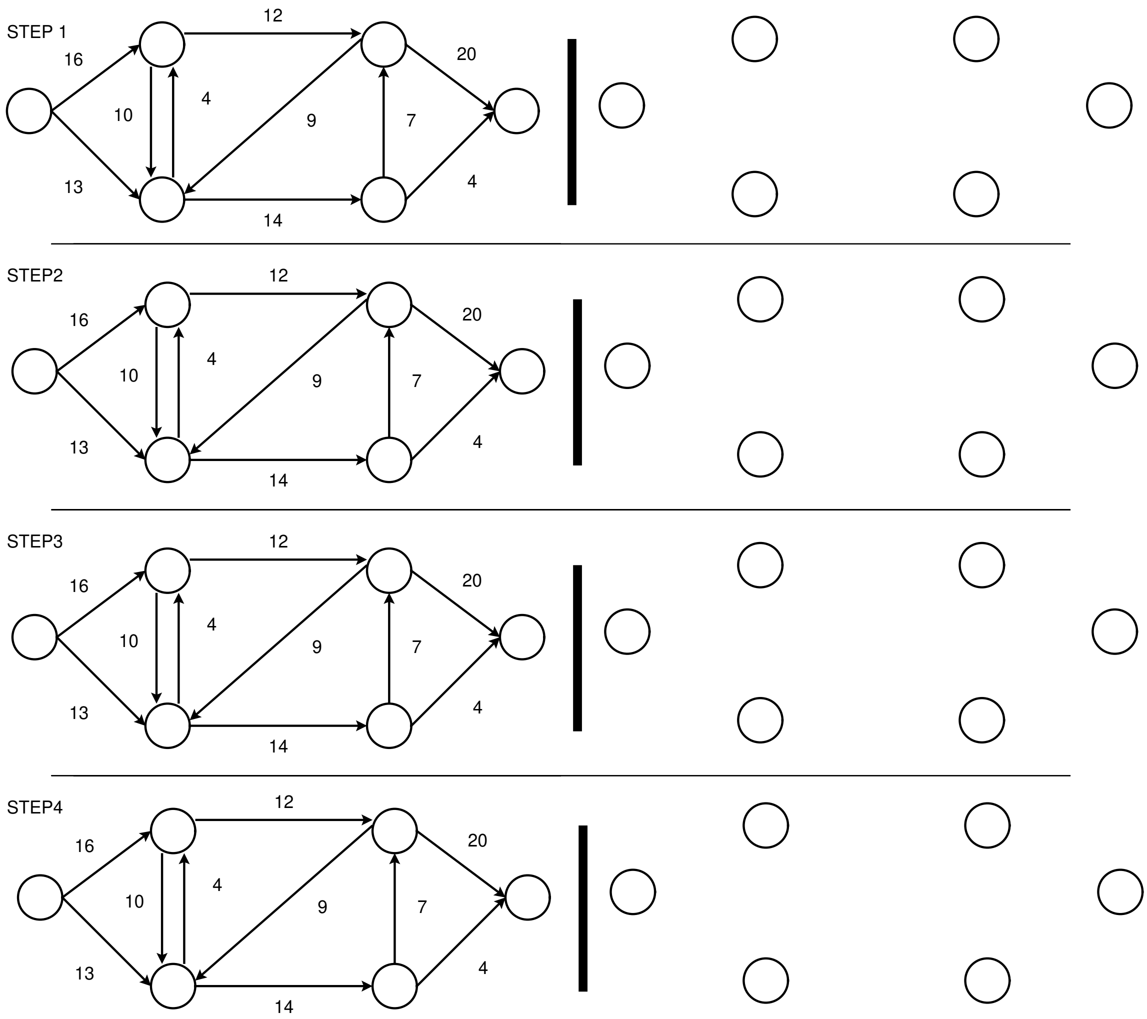
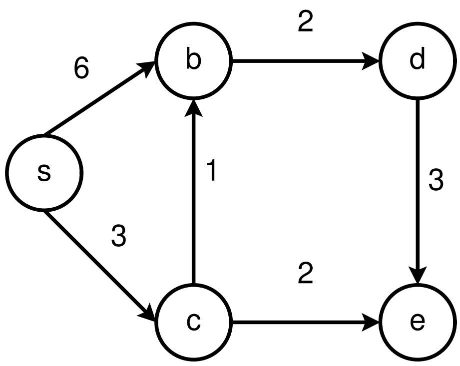
e) Illustrate step-by-step the execution of the Ford-Fulkerson method (only the first four steps). Show in the left column the flow of the network. Illustrate the corresponding residual network in the right column and mark the augmenting path you select. (Hint: draw backward and forward edges in the residual network).

Left column: flow/capacity Right column: residual network



Problem 4 (15 points) Let $G = (V, E)$ be a directed, weighted graph. Let $s \in V$ be the start vertex. Consider the example:



a) Give the adjacency list representation of this graph.

b) Give the adjacency matrix representation of this graph.

c) In addition of the graph, what other data structure is maintained by Dijkstra's single source shortest path algorithm?

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Problem 5 (15 points) Consider a binary counter. The basic operation is incrementing the counter by 1, beginning at 0. The complexity is the number of counter bits that need to be changed.

a) Make a worst case analysis for counting up to n in terms of the complexity measure above.

b) List the accumulated number of carries when counting up to 10.

c) Determine the amortized complexity of counting up to n. Explain in detail.

Problem 6 (15 points) Suppose that a hash function is given by

$$h(m) = 4m + 3 \bmod 9.$$

We consider a hashing scheme that is based on the hash function h. Collisions are resolved by chaining. List insertion is done onto the front of the list.

a) Illustrate the hash table after inserting the elements

$$212, 17, 32, 14, 8, 4, 31.$$

b) How many elements have to be inserted into this hash table so that the expected number of collisions is at least one? Assume simple uniform hashing. Give a complete argument.