Problem Set 8

CPSC 311 Analysis of Algorithms

Andreas Klappenecker

The assignment is due next Monday, December 5, before class.

Solve the following exercises from the textbook; note that the book distinguishes between problems and exercises.

1. Make sure that you fully understand the problems SAT, 3-CNF-SAT=3-SAT, CLIQUE, VERTEX-COVER, HAM-CYCLE, TSP, SUBSET-SUM, INDEPENDENT SET (cf. Problem 34-1), k-COLOR (cf. Problem 34-3).

2. Let ϕ be a boolean formula in 3-CNF. An $\neq$-assignment to the variables of ϕ is one where each clause contains two literals with unequal truth values. In other words, an $\neq$-assignment satisfies ϕ without assigning three true literals in any clause.
   (a) Show that the negation of a $\neq$-assignment to ϕ is also a $\neq$-assignment.
   (b) Let $\neq$SAT be the collection of boolean formulas in 3-CNF that have an $\neq$-assignment. Show that $3-\text{SAT} \leq_P \neq \text{SAT}$, and that $\neq$SAT is NP-complete. [Hint: Replace each clause $(y_1 \vee y_2 \vee y_3)$ by two clauses $(y_1 \vee y_2 \vee z_i)$ and $(\neg z_i \vee y_3 \vee b)$, where z_i is a new varable for each clause and b is a single additional new variable.]

3. Problem 34-3 d,e,f (that is, prove that 3-SAT$\leq_P$ 3-COLOR).

4. Problem 34-1

Make sure that you structure your answers well. Please typeset your solutions in LaTeX or turn in a neatly written solution.