

**Problem Set 1**  
CPSC 289 Discrete Structures  
Andreas Klappenecker

**The assignment is due on Friday, September 5, before class.**

Construct a truth table for each of the following compound propositions. Each truth table should also include all subformulas of the given formula. For example, constructing the truth table of  $\neg p \vee \neg q$  should list the truth values for  $p$ ,  $q$ ,  $\neg p$ ,  $\neg q$ , and  $\neg p \vee \neg q$ .

1.  $((p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p))$
2.  $((p \rightarrow q) \rightarrow (q \rightarrow r)) \rightarrow (p \rightarrow r)$
3.  $((p \leftrightarrow q) \oplus (p \leftrightarrow \neg q))$
4.  $((p \wedge q) \vee (p \rightarrow \neg q)) \leftrightarrow (p \oplus q)$

The remaining exercises are mostly taken from the textbook.

5. Section 1.1, Exercise 42. Argue carefully.
6. Prove the logical equivalences given in Table 8 on page 25 of the textbook using truth tables.
7. Section 1.2, Exercise 22.
8. For propositions  $p$  and  $q$ , define  $p \mid q$  to be true if and only if not both  $p$  and  $q$  are true. Give logically equivalent formulations of  $\neg p$ ,  $p \wedge q$ ,  $p \vee q$ ,  $p \oplus q$ ,  $p \rightarrow q$ ,  $p \leftrightarrow q$  using compound propositions that involve only  $\mid$ ,  $p$  and  $q$  (find terms that are as simple as possible). Prove your results using truth tables.
9. Is the connective  $\mid$  associative, that is, is  $((p \mid q) \mid r)$  logically equivalent to  $(p \mid (q \mid r))$ ? Prove this or find a counter example.
10. Prove by contradiction that 57 is an odd integer. Use the following facts: (i) An even integer is a multiple of 2; (ii) The integers are totally ordered by the less or equal relation  $\leq$ . (iii) If  $a$ ,  $b$ , and  $c$  are integers such that  $a \leq b$  and  $c > 0$ , then  $ac \leq bc$ . Do not use the fact that  $57/2 = 28\frac{1}{2}$ ; your argument should only use the above properties of the integers.

Typeset your answers or neatly print your answers. This is a good opportunity to learn L<sup>A</sup>T<sub>E</sub>X!

**Read chapter 1 in the textbook. Skim through Section 4.1. Read all lecture notes.** Use paper and pencil while reading through this material. When you read a definition, make sure that you completely understand it.