Challenge Problem 2

CPSC 489/689 Quantum Algorithms

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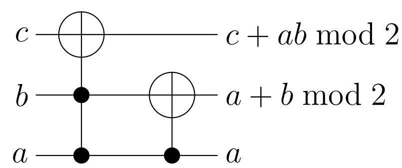
The following quantum circuit represents a half-adder; it calculates the sum $a + b \bmod 2$, and the carry ab of the inputs a and b:



The circuit implements a unitary matrix U_{add}, which is determined by

$$\begin{array}{rclcrcl}
U_{add}|000\rangle & = & |000\rangle, & \qquad & U_{add}|100\rangle & = & |100\rangle, \\
U_{add}|001\rangle & = & |011\rangle, & & U_{add}|101\rangle & = & |111\rangle, \\
U_{add}|010\rangle & = & |010\rangle, & & U_{add}|110\rangle & = & |110\rangle, \\
U_{add}|011\rangle & = & |101\rangle, & & U_{add}|111\rangle & = & |001\rangle.
\end{array}$$

Let $m(U)$ denote the minimal number of controlled-not and single qubit gates, which are needed to realize $U \in \mathcal{U}(2^n)$. The challenge is to determine $m(U_{add})$. In other words, how many controlled-not gates and single qubits gates are needed in an optimal implementation of U_{add}? You need to prove your result.

Remark. Let T denote the unitary matrix corresponding to the Toffoli gate. Notice that $|m(T) - m(U_{add})| \leq 1$.

I offer a **Challenges in Quantum Computing Award**, worth US\$ 100, for the first correct solution to this problem.

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