Problem Set 6
CPSC 629 Analysis of Algorithms
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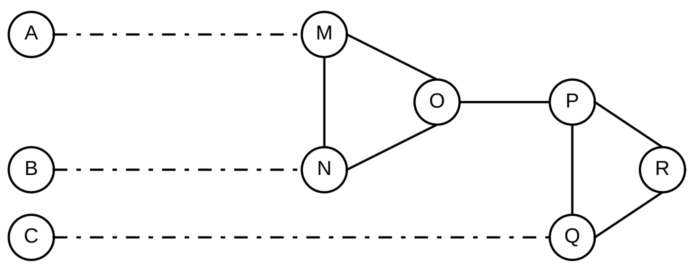
The assignment is due Tuesday (12/03/2002), before class.

A graph $G = (V, E)$ is called 3-colorable if and only if it is possible to label the vertices of G with t, f, or d, such that no two vertices with the same label are connected by an edge in E.

Q1 Consider the following graph. It is easy to see that this graph is 3-colorable.



Assume that the vertices A, B, C are assigned the label t or f. When is it possible to label O and R with t in a 3-coloring of this graphwith t, f, and d?

Q2 Suppose you are given another triangle (a clique with three vertices) such that the nodes are labeled with t, f and d. How can you connect this triangle to the graph given in Q1 such that the labels of A, B, C, and R are either t or f?

Q3 Show that 3-colorability of a graph is NP-complete by giving a polynomial reduction from 3SAT. In other words, given a boolean formula $p(x)$ in 3-CNF with n variables and m clauses, show how to define a graph that is 3-colorable if and only if $p(x)$ is satisfiable. Make sure to explain the following:
(a) Why is 3COLOR in NP?
(b) How many vertices are needed in your method?
(c) How the literals are encoded.
(d) Why your method works.
Hint: Use the gadgets given in Q1 and Q2.

Q4 Give the graph that is associated with $(x_1 \vee \overline{x}_2 \vee x_3) \wedge (\overline{x}_1 \vee \overline{x_3} \vee x_4)$ to illustrate your method, and explain.