Little Dictionary of Generating Functions

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Notations: $[S(n)] = 1$ if the predicate $S(n)$ is true, 0 otherwise.
$[m\backslash n] = [m \mid n] = [m \text{ divides } n]$

Sequence	Generating Function	Closed Form
$(1, 0, 0, \ldots)$	$\sum_{n\geq 0} [n = 0] z^n$	1
$(0, \ldots, 0, 1, 0, 0, \ldots)$	$\sum_{n\geq 0} [n = m] z^n$	z^m
$(1, 1, 1, \ldots)$	$\sum_{n\geq 0} z^n$	$\dfrac{1}{1-z}$
$(1, -1, 1, -1, \ldots)$	$\sum_{n\geq 0} (-1)^n z^n$	$\dfrac{1}{1-z}$
$(1, 0, 1, 0, \ldots)$	$\sum_{n\geq 0} z^{2n}$	$\dfrac{1}{1-z^2}$
$(1, 2, 3, 4, 5, \ldots)$	$\sum_{n\geq 0} (n+1) z^n$	$\dfrac{1}{(1-z)^2}$
$(1, 2, 4, 8, 16, \ldots)$	$\sum_{n\geq 0} 2^n z^n$	$\dfrac{1}{1-2z}$
$(1, 4, 6, 4, 1, 0, 0, \ldots)$	$\sum_{n\geq 0} \binom{4}{n} z^n$	$(1+z)^4$
$(1, c, \binom{c}{2}, \binom{c}{3}, \ldots)$	$\sum_{n\geq 0} \binom{c}{n} z^n$	$(1+z)^c$
$(1, c, \binom{c+1}{2}, \binom{c+2}{3}, \ldots)$	$\sum_{n\geq 0} \binom{c+n-1}{n} z^n$	$\dfrac{1}{(1-z)^c}$
$(1, 1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \ldots)$	$\sum_{n\geq 0} \dfrac{1}{n!} z^n$	e^z
$(0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots)$	$\sum_{n\geq 1} \dfrac{1}{n} z^n$	$\ln \dfrac{1}{1-z}$
$(0, 1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \ldots)$	$\sum_{n\geq 1} \dfrac{(-1)^{n+1}}{n} z^n$	$\ln(1+z)$

Memorize these basic sequences.