Joint Decoding of Content-Replication Codes for Flash Memories

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I. Introduction

Flash memories are attractive due to their superior performances over prior mass storage technologies. However, one challenge is the data reliability as several types of noise [1] exist. One technology to combat errors is a strong error correcting code, e.g., LDPC code. Another mechanism is *memory scrubbing* [2], i.e., while errors accumulate in a codeword, with the next block erasure, the codeword is corrected and a new error-free codeword is written back to the memory. However, in flash memory rewrites are made in an *out-of-place* fashion, i.e., an updated codeword is stored at a new physical address and the original codeword remains in the memory. Those mechanisms lead to multiple copies of codewords, i.e., the *content-replicated codewords* problem.

In this work, we enhance the flash memory reliability by utilizing the existence of two content-replicated codewords for decoding, including an old codeword and a new codeword storing the same information. We aim at designing a *joint decoder* having access to both content-replicated codewords, and explore its decoding performance. This leads to reliability improvement in flash memories. We further study a new paradigm where the two content-replicated codewords have different forms for better performance.

II. Problem Statement

Let $\mathcal{D} = \{0, 1, \cdots, M-1\}$ be the message set for $M \in \mathbb{N}$, and let $\mathcal{X}$ and $\mathcal{Y}$ be two alphabets of the symbols stored in a cell. Let two encoders be $f_1 : \mathcal{D} \to \mathcal{X}^N$ and $f_2 : \mathcal{D} \to \mathcal{X}^N$, and the desired joint decoder be $h : \mathcal{Y}^N \times \mathcal{Y}^N \to \mathcal{D}$, where N is the length of codewords. Let $\mathbb{P} = (\mathcal{X}, \mathcal{Y}, \mathcal{P}_{Y|X})$ and $\mathbb{Q} = (\mathcal{X}, \mathcal{Y}, \mathcal{Q}_{Y|X})$ be two independent channels.

We illustrate the model in Fig 1. Here, m is a common message to both encoders, the N-dimensional vectors $x_0^{N-1}(1)$, $x_0^{N-1}(2) \in \mathcal{X}^N$ are two codewords obtained through two encoders (those encoders are not necessarily identical), and $y_0^{N-1}(1)$, $y_0^{N-1}(2)$ are two noisy codewords through $\mathbb{P}$ and $\mathbb{Q}$. The task is to design a joint decoder to *reliably* estimate the message m, $\hat{m}$, giving $y_0^{N-1}(1)$ and $y_0^{N-1}(2)$.

The problem statement is presented below:

Definition 1. Given a $(N, 2^{NR})$ code, $\mathbb{P}$ and $\mathbb{Q}$, $\mathcal{D} = \{0, 1, \cdots, 2^{NR} - 1\}$, $f_1 : \mathcal{D} \to \mathcal{X}^N$ and $f_2 : \mathcal{D} \to \mathcal{X}^N$, the task is to design a joint decoding function $h : \mathcal{Y}^N \times \mathcal{Y}^N \to \mathcal{D}$ such that $Pr(h(y_0^{N-1}(1), y_0^{N-1}(2)) \neq i | x_0^{N-1}(1) = f_1(i), x_0^{N-1}(2) = f_2(i))) \to 0$ for $i \in \mathcal{D}$ as $N \to \infty$.

III. Solutions

For simplicity, assume $\mathbb{P}$ and $\mathbb{Q}$ are Binary Erasure Channels with the same parameter ϵ, and both encoders are LDPC encoders. We use the following notations for our LDPC codes: let the rate of two LDPC codes be $\frac{K}{N}$, let $\mathbf{G}_1$, $\mathbf{G}_2$ be the encoding matrices, and $\mathbf{H}_1$, $\mathbf{H}_2$ denotes their parity check matrices. Let $y_0^{N-1}(1), y_0^{N-1}(2) \in \{0, 1, ?\}^N$ be two codewords received.

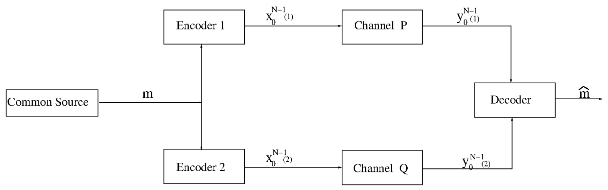

Fig. 1. Illustration of joint decoding content-replicated codewords.

All proposed solutions follow the same outline, i.e., construct a LDPC code for the joint decoder based on the two LDPC codes used, determine its parity check matrix, and apply the conventional belief propagation decoder.

1) Joint decoder of identical content-replicated codes: The given encoding functions are *identical* in this case, i.e., $\mathbf{G}_1 = \mathbf{G}_2$ and $\mathbf{H}_1 = \mathbf{H}_2$.

Given $y_0^{N-1}(1)$ and $y_0^{N-1}(2)$, a codeword y_0^{N-1} for the joint decoder is obtained by comparing $y_0^{N-1}(1)$ and $y_0^{N-1}(2)$ to further eliminate erasures, i.e., for $i = 0, 1, \cdots, N-1$, $y_i = \begin{cases} ? & \text{if } y_i(1) = y_i(2) = ?, \\ y_i(1) = y_i(2) & \text{otherwise.} \end{cases}$
The parity check matrix for y_0^{N-1} is $\mathbf{H}_1$. The decoding result is obtained by applying belief propagation to y_0^{N-1} with $\mathbf{H}_1$ and initial erasure probability ϵ^2.

Let $\lambda(x)$ and $\rho(x)$ be degree distributions for the LDPC codes used, let $\epsilon^{BP}(\lambda, \rho)$ be its conventional threshold, and let $\epsilon^{BP}_{iden}(\lambda, \rho)$ denote the threshold for our joint decoder. The comparison of $\epsilon^{BP}_{iden}(\lambda, \rho)$ and $\epsilon^{BP}(\lambda, \rho)$ is presented in Table I, and we have $\epsilon^{BP}_{iden} > \epsilon^{BP}$.

2) Joint decoder of content-replicated codes with identical information bits: The encoding functions are *different* in this case, i.e., $\mathbf{G}_1 \neq \mathbf{G}_2$ and $\mathbf{H}_1 \neq \mathbf{H}_2$, but the codewords carry identical information bits when regarding them as systematic codes, that is, two encoding functions are $x_0^{N-1}(1) = u_0^{K-1}\mathbf{G}_1$ and $x_0^{N-1}(2) = u_0^{K-1}\mathbf{G}_2$.

Let $\mathcal{I}_1, \mathcal{I}_2 \subseteq \{0, 1, \cdots, N-1\}$ be the information bit index sets for $y_0^{N-1}(1)$ and $y_0^{N-1}(2)$, and let $\mathcal{P}_1$ and $\mathcal{P}_2$ be their parity check bit index sets. Let $y_0^{N-1}(1)_{\mathcal{I}_1} = (y_i(1) : i \in \mathcal{I}_1)$, i.e., information bits of $y_0^{N-1}(1)$, and similar notations apply to $y_0^{N-1}(1)_{\mathcal{P}_1}$ and $y_0^{N-1}(2)_{\mathcal{P}_2}$. Let $g(\cdot) : \mathcal{I}_1 \to \mathcal{I}_2$ be a one-to-one mapping. Similarly, further erasure elimination is possible for $y_0^{N-1}(1)_{\mathcal{I}_1}$ when comparing it with $y_0^{N-1}(2)_{\mathcal{I}_2}$, that is we define $(y_0^{N-1})_{\mathcal{I}_1}$, where $y_i = \begin{cases} ? & \text{if } y_i(1) = y_{g(i)}(2) = ?, \\ y_i(1) & \text{otherwise.} \end{cases}$
Then, a constructed codeword is $y_0^{2N-K-1} = [(y_0^{N-1})_{\mathcal{I}_1}, y_0^{N-1}(1)_{\mathcal{P}_1}, y_0^{N-1}(2)_{\mathcal{P}_2}]$. That is, y_0^{2N-K-1} is constructed by extracting information bits from $y_0^{N-1}(1)$ and $y_0^{N-1}(2)$, and appending parity check bits from $y_0^{N-1}(1)$ and $y_0^{N-1}(2)$. An example is illustrated in Fig. 2.

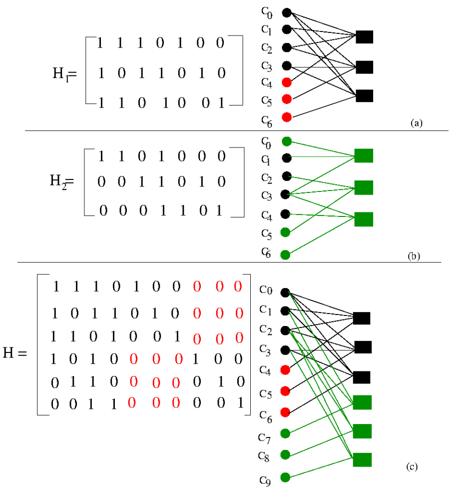

Fig. 2. Illustration of constructed y_0^{2N-K-1} and $\mathbf{H}$. (a) The Tanner graph and $\mathbf{H}_1$ for $y_0^{N-1}(1)$, where information bits are black and parity check bits are red; (b) The Tanner graph and $\mathbf{H}_2$ for $y_0^{N-1}(2)$, where information bits are black and parity check bits are green; (c) The constructed Tanner graph and $\mathbf{H}$ based on (a) and (b), where information bits are black, parity check bits from $y_0^{N-1}(1)$ are red, and parity check bits from $y_0^{N-1}(2)$ are green.

Let $\mathbf{H}_1 = [\mathbf{H}_{1,0}, \mathbf{H}_{1,1}, \cdots, \mathbf{H}_{1,N-1}]$, let $\mathbf{H}_{1,\mathcal{I}_1} = [\mathbf{H}_{1,i} : i \in \mathcal{I}_1]$, and let $\mathbf{H}_{1,\mathcal{P}_1} = [\mathbf{H}_{1,i} : i \in \mathcal{P}_1]$. Similarly, we divide $\mathbf{H}_2$ into $\mathbf{H}_{2,\mathcal{I}_2}$ and $\mathbf{H}_{2,\mathcal{P}_2}$. Then, the parity check matrix $\mathbf{H}$ for y_0^{2N-K-1} is of the form in Fig 3. An example is illustrated in Fig. 2.

The decoding result is obtained by applying belief propagation to y_0^{2N-K-1} with $\mathbf{H}$, the initial erasure probability ϵ^2 for $(y_0^{N-1})_{\mathcal{I}_1}$, and ϵ for $y_0^{N-1}(1)_{\mathcal{P}_1}$ and $y_0^{N-1}(2)_{\mathcal{P}_2}$.

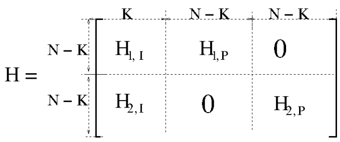

Fig. 3. Illustration of the parity check matrix $\mathbf{H}$

Let $\epsilon_{dif}^{BP}(\lambda, \rho)$ be the joint decoder threshold when the two LDPC codes have the same distribution functions (λ, ρ). We can prove that $\epsilon_{dif}^{BP}(\lambda, \rho) = \sup\{\epsilon \in [0,1] : x_\infty(x) = 0\}$, where $x_{l+1}(x) = f_i(\epsilon, x_l(x), y_l(x))$, $y_{l+1} = f_p(\epsilon, x_l(x), y_l(x))$, $x_0(x) = x^2$ and $y_0(x) = x$ for defined functions f_i and f_p. (Due to space limitation, we omit its proof here.) We present several $\epsilon^{BP}, \epsilon_{iden}^{BP}, \epsilon_{dif}^{BP}$ in Table I, from which we see that $\epsilon_{iden}^{BP} > \epsilon_{dif}^{BP} > \epsilon^{BP}$.

TABLE I
COMPARISON OF $\epsilon^{BP}, \epsilon_{iden}^{BP}$ AND ϵ_{dif}^{BP}

(d_v, d_c)	ϵ^{BP}	ϵ_{iden}^{BP}	ϵ_{dif}^{BP}
(3,4)	0.6474	0.8046	0.7549
(3,5)	0.5176	0.7194	0.6807
(3,6)	0.4294	0.6553	0.6270
(4,6)	0.5061	0.7114	0.6285
(4,8)	0.3834	0.6192	0.5581

3) Joint decoder of content-replicated codes with transformed information bits: The two encoding functions are *related* in this case. More specially, let $\mathbf{G}_3$ be an *intermediate* LDPC generator matrix with the rate $\frac{1}{2}$. Similarly, let $\mathcal{I}_i$ and $\mathcal{P}_i$ denote the information bit index set and parity check bit index set for codes with $\mathbf{G}_i$, $i = 1, 2, 3$.

The encoding algorithm is below, where $(x_0^{N-1})_{\mathcal{P}_3}$ denotes the subvector $(x_i : i \in \mathcal{P}_3)$.

1) f_1: $x_0^{N-1}(1) = u_0^{K-1}\mathbf{G}_1$.
2) $v_0^{K-1} = (u_0^{K-1}\mathbf{G}_3)_{\mathcal{P}_3}$.

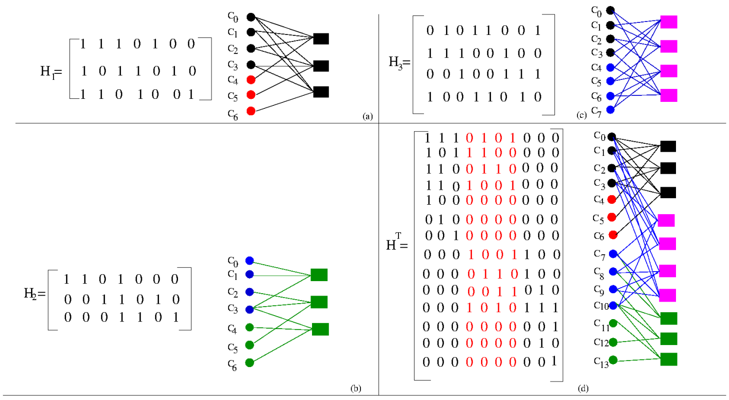

Fig. 4. Illustration of constructed y_0^{2N-1} and $\mathbf{H}$. (a) The Tanner graph and $\mathbf{H}_1$ for $y_0^{N-1}(1)$, where information bits are black and parity check bits are red; (b) The Tanner graph and $\mathbf{H}_2$ for $y_0^{N-1}(2)$, where information bits are green and parity check bis are blue; (c) The Tanner graph and $\mathbf{H}_3$ for v_0^{K-1}, where information bits are black and parity check bits are blue; (d) The constructed Tanner graph and $\mathbf{H}$ for y_0^{2N-1}.

3) f_2: $x_0^{N-1}(2) = v_0^{K-1}\mathbf{G}_2$.

That is, $(x_0^{N-1}(1))_{\mathcal{I}_1}$ and $(x_0^{N-1}(2))_{\mathcal{I}_2}$ are related through $\mathbf{G}_3$. Refer to Fig. 4 for an example.

A decoding codeword for the joint decoder is obtained by assembling $y_0^{N-1}(1)$ and $y_0^{N-1}(2)$ in the following way, $y_0^{2N-1} = (y_0^{N-1}(1)_{\mathcal{P}_1}, y_0^{N-1}(1)_{\mathcal{I}_1}, y_0^{N-1}(2)_{\mathcal{P}_2}, y_0^{N-1}(2)_{\mathcal{I}_2})$.

Let $\mathbf{H}_3$ be parity check matrices corresponding to $\mathbf{G}_3$. Then, the parity check matrix $\mathbf{H}$ for y_0^{2N-1} is of the form in Fig. 5. An example is presented in Fig. 4.

The decoding result is obtained by applying belief propagation to y_0^{2N-1} with $\mathbf{H}$ and initial erasure probability ϵ.

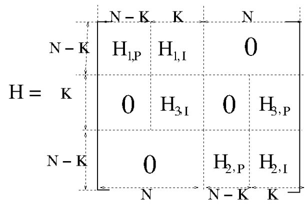

Fig. 5. Illustration of parity check matrix $\mathbf{H}$

Let ϵ_{cou}^{BP} be the threshold of joint decoder in this case. We calculate several ϵ_{cou}^{BP} in Table II, where the first row indicates the regular LDPC for $\mathbf{G}_3$, and the first column indicates the regular LDPC code for $\mathbf{G}_1$ and $\mathbf{G}_2$. From this table, we see that $\epsilon_{cou}^{BP} > \epsilon_{iden}^{BP} > \epsilon_{dif}^{BP} > \epsilon^{BP}$ is possible with appropriate $\mathbf{G}_3$.

TABLE II
CALCULATION OF ϵ_{cou}^{BP}

(d_v, d_c)	(1,2)	(2,4)	(3,6)	(4,8)
(3,4)	0.7549	0.7975	0.7823	0.7697
(3,5)	0.6807	0.7555	0.7367	0.7163
(3,6)	0.6270	0.7295	0.711	0.6854
(4,6)	0.6285	0.7108	0.7001	0.6762
(4,8)	0.5581	0.6855	0.6875	0.6479

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