CSCE 636 Neural Networks (Deep Learning)

Lecture 12: Deep Reinforcement Learning (continued)

Anxiao (Andrew) Jiang

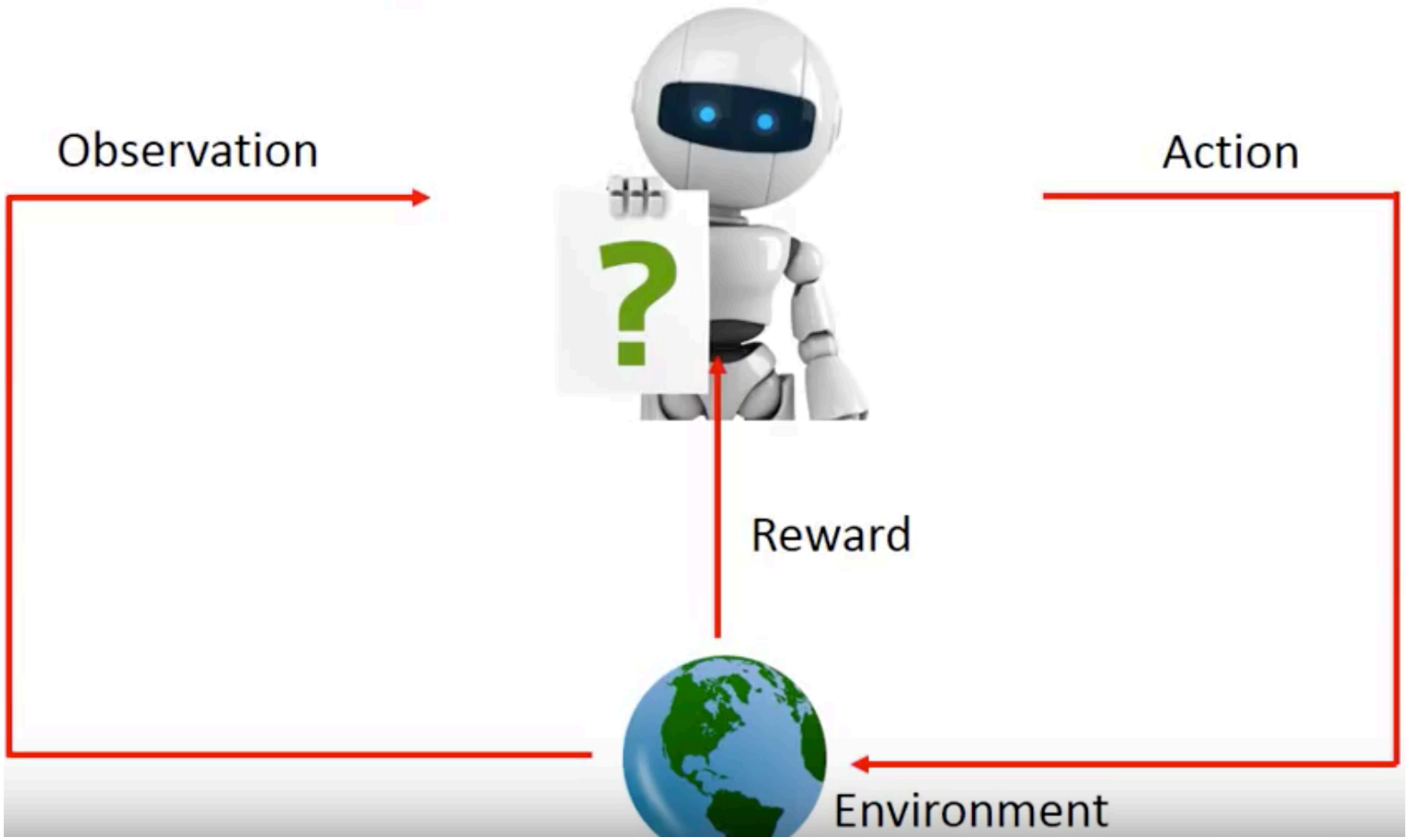
Based on the interesting lecture of Prof. Hung-yi Lee "Deep Reinforcement Learning"
https://www.youtube.com/watch?v=W8XF3ME8G2I&list=PLJV_el3uVTsPy9oCRY30oBPNLCo89yu49&index=33

Policy-based Approach

Learning an Actor

Note: Actor means “Agent”

Machine Learning
≈ Looking for a Function



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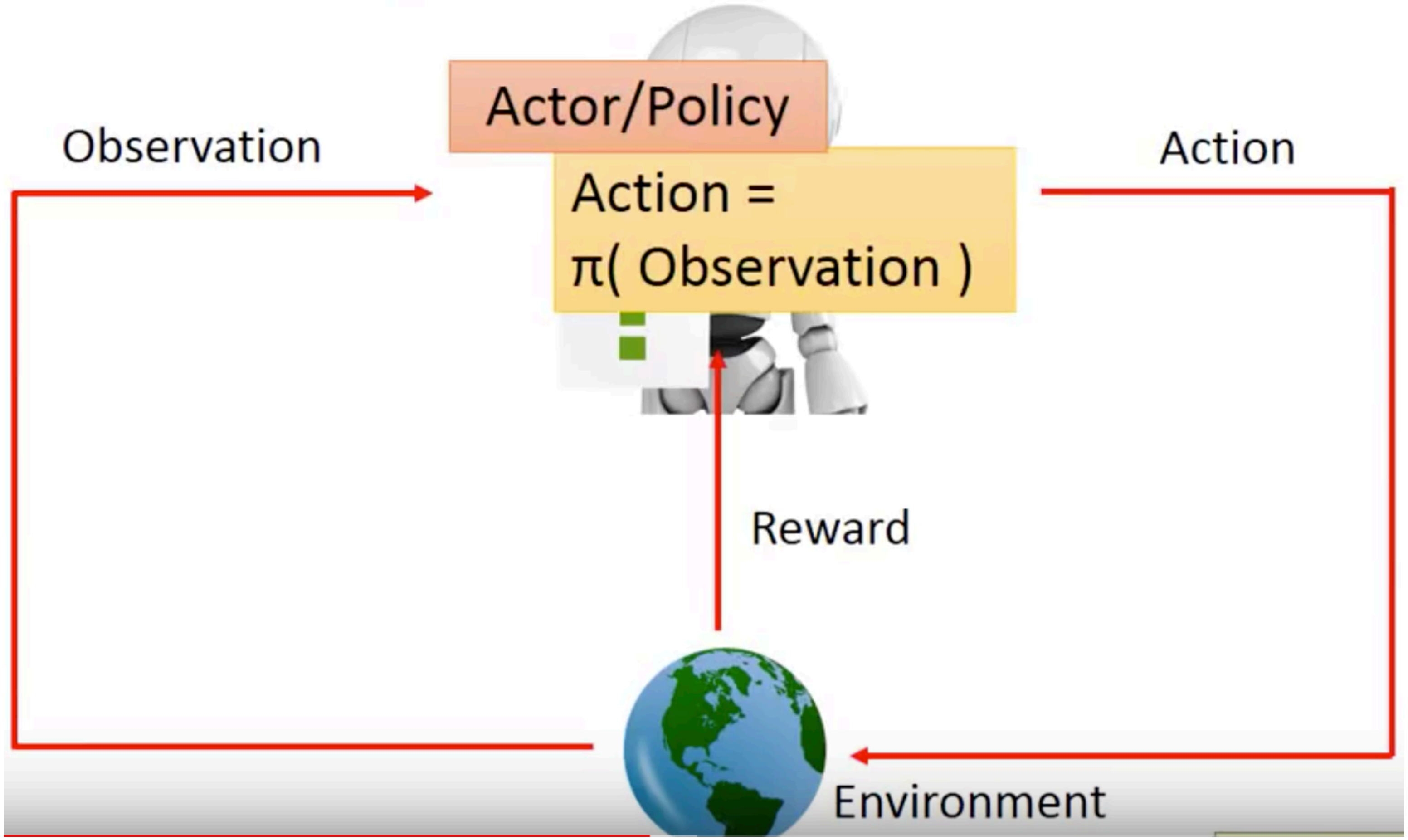

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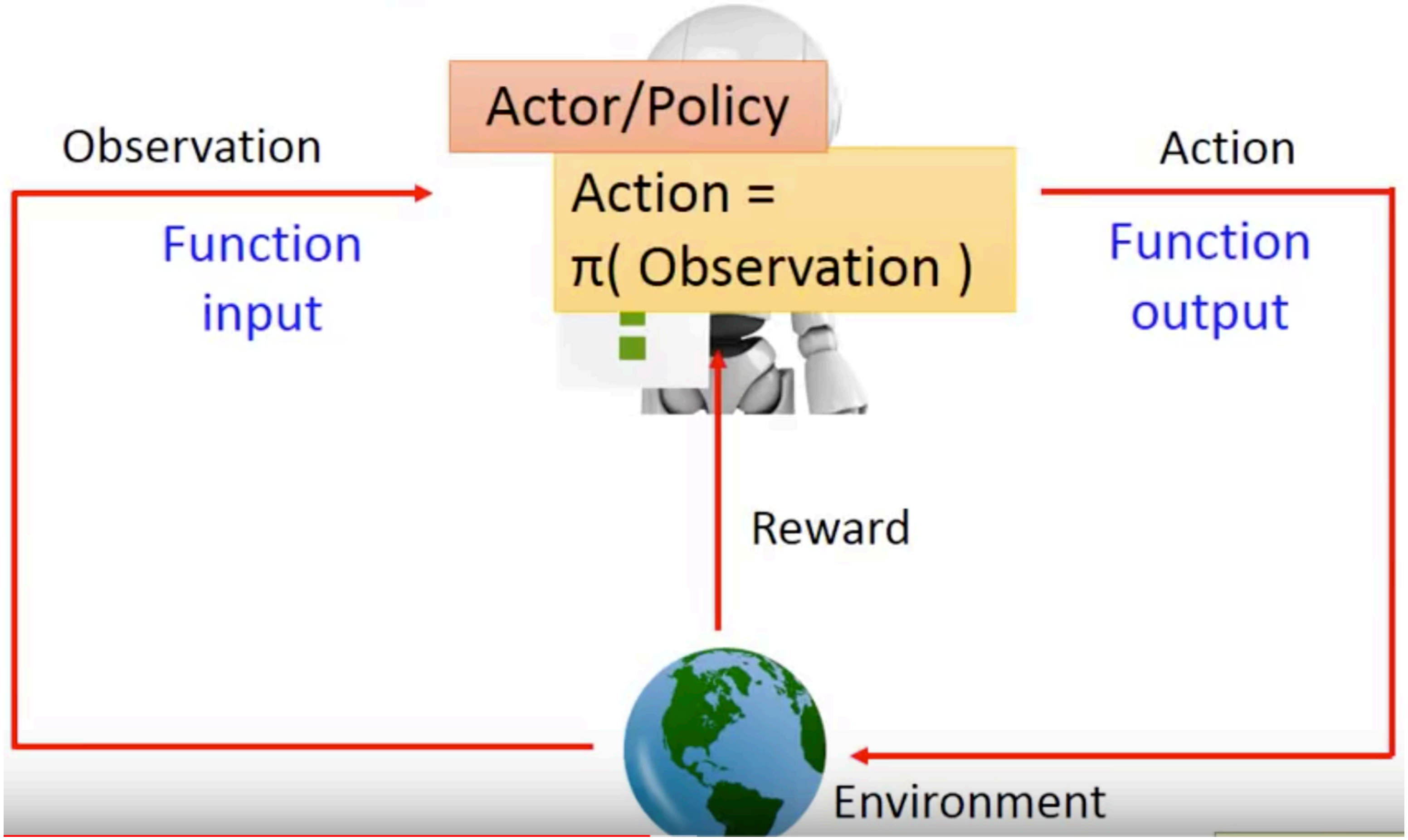

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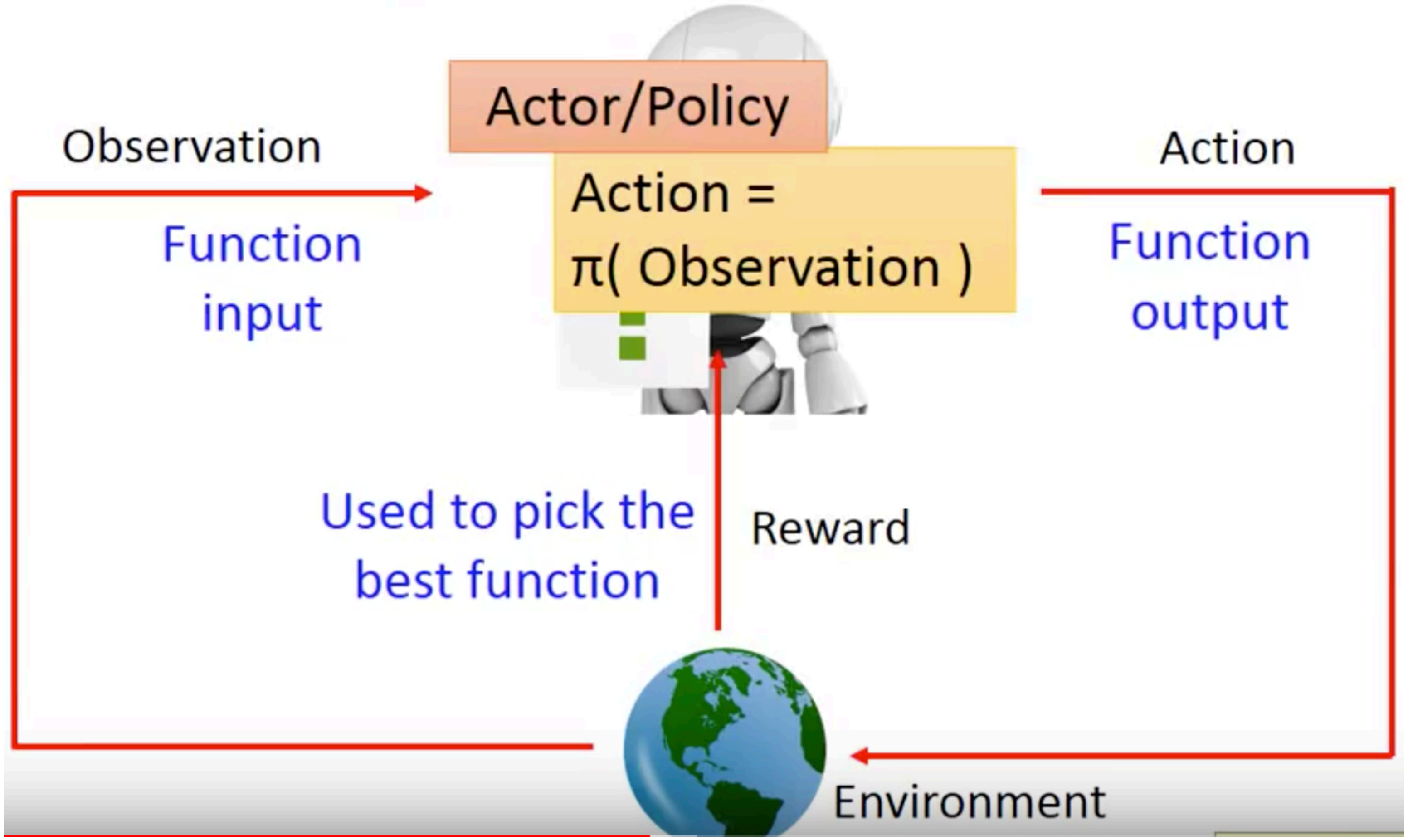

Neural network as Actor

- Input of neural network: the observation of machine represented as a vector or a matrix

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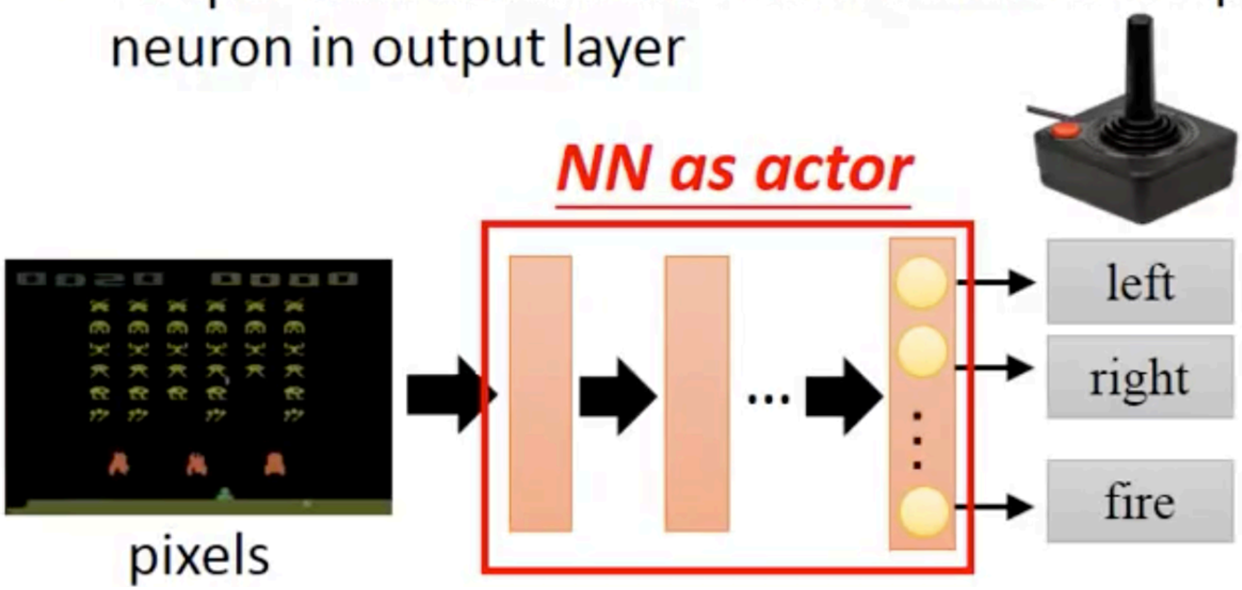

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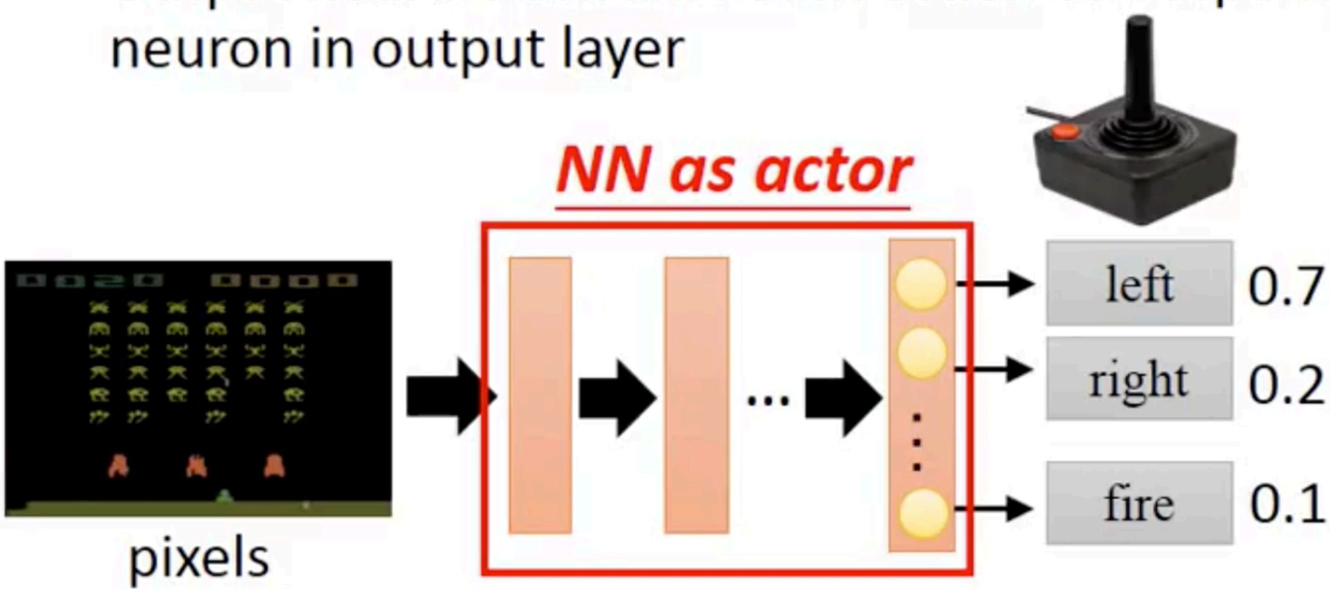

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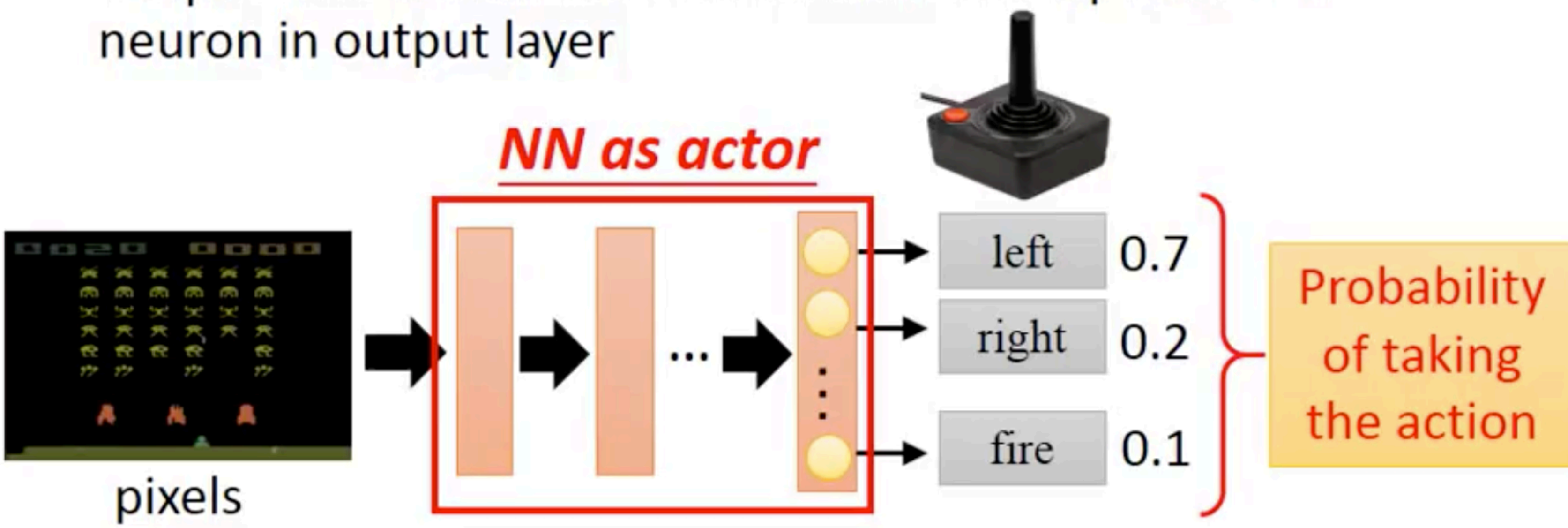
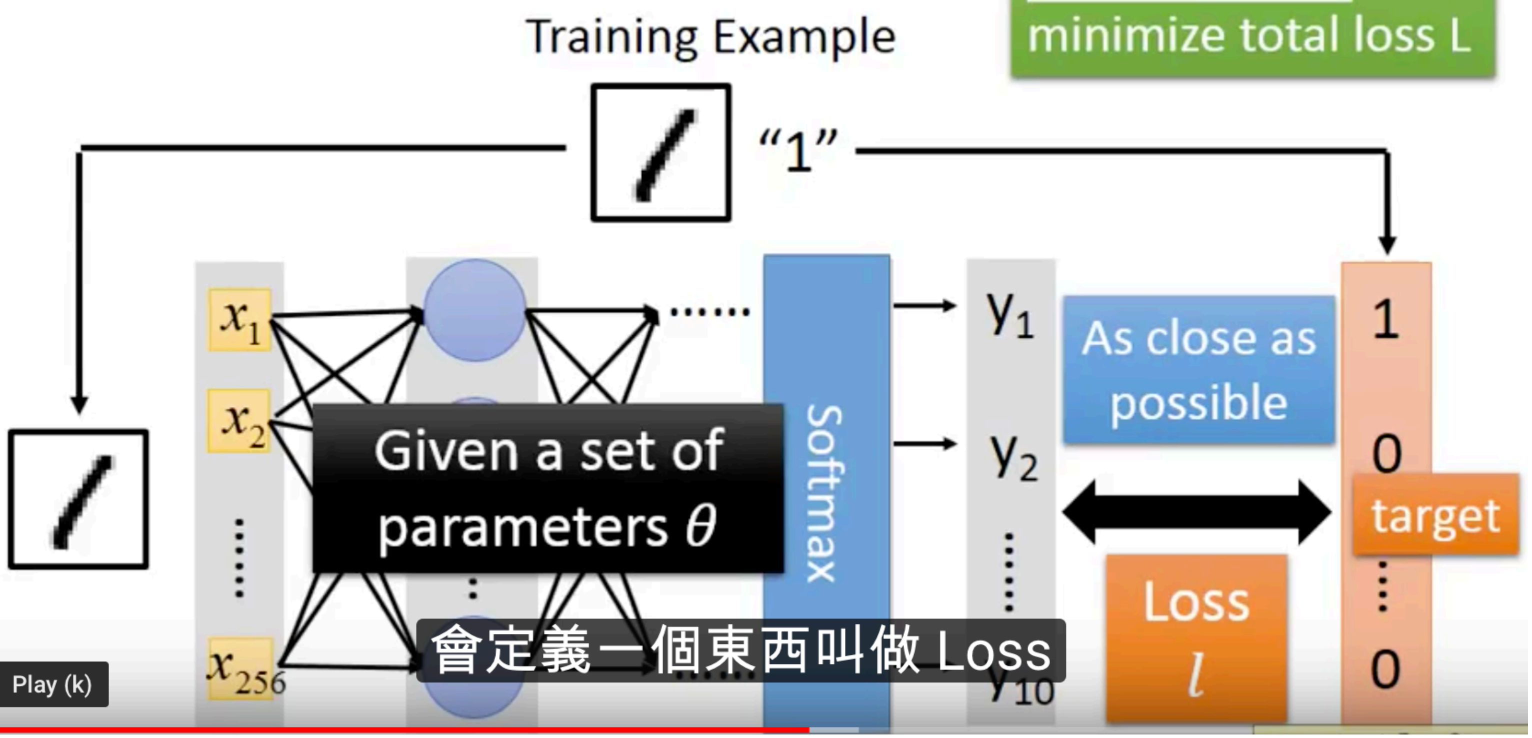


Goodness of Actor

Total Loss: $L = \sum_{n=1}^{N} l_n$

- Review: Supervised learning

Find ***the network parameters θ^**** that minimize total loss L



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  - Start with observation s_1
  - Machine decides to take a_1
  - Machine obtains reward r_1
  - Machine sees observation s_2
  - Machine decides to take a_2
  - Machine obtains reward r_2
  - Machine sees observation s_3
  - ……
  - Machine decides to take a_T
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Total reward: $R_\theta = \sum_{t=1}^{T} r_t$

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We define $\bar{R}_\theta$ as the <u>*expected value*</u> of R_θ

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$\bar{R}_\theta$

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Sum over all possible trajectory

Use π_θ to play the game N times, obtain $\{\tau^1, \tau^2, \cdots, \tau^N\}$

Sampling τ from $P(\tau|\theta)$ N times

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Gradient Ascent

- Problem statement

$$\theta^* = arg\max_{\theta} \bar{R}_\theta \qquad \bar{R}_\theta = \sum_{\tau} R(\tau)P(\tau|\theta)$$

- Gradient ascent
  - Start with θ^0
  - $\theta^1 \leftarrow \theta^0 + \eta\nabla\bar{R}_{\theta^0}$
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$$\nabla \bar{R}_\theta$$

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$$\nabla \bar{R}_\theta = \begin{bmatrix} \partial \bar{R}_\theta / \partial w_1 \\ \partial \bar{R}_\theta / \partial w_2 \\ \vdots \\ \partial \bar{R}_\theta / \partial b_1 \\ \vdots \end{bmatrix}$$

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It can even be a black box.

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怎麼計算 log * P(τ | θ) 的 gradient

Gradient Ascent

$\nabla log P(\tau|\theta) = ?$

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$p(s_1)p(a_1|s_1,\theta)p(r_1,s_2|s_1,a_1)p(a_2|s_2,\theta)p(r_2,s_3|s_2,a_2)\cdots$

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$$= p(s_1)\prod_{t=1}^{T} p(a_t|s_t,\theta)p(r_t,s_{t+1}|s_t,a_t)$$

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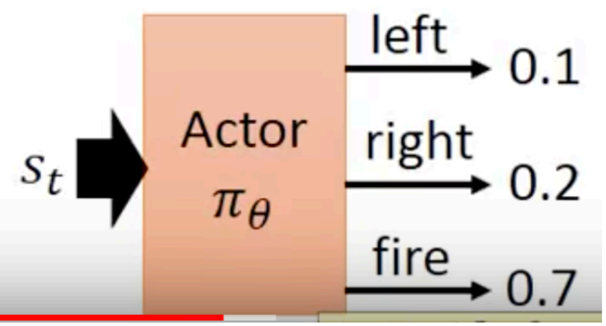
$$= \underbrace{p(s_1)}_{\text{not related to your actor}} \prod_{t=1}^{T} \underbrace{p(a_t|s_t,\theta)}_{\text{Control by your actor } \pi_\theta} \underbrace{p(r_t,s_{t+1}|s_t,a_t)}_{\text{not related to your actor}}$$

not related to your actor

Control by your actor π_θ



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$$\nabla logP(\tau|\theta) = \sum_{t=1}^{T} \nabla logp(a_t|s_t,\theta)$$

$$\theta^{new} \leftarrow \theta^{old} + \eta \nabla \bar{R}_{\theta^{old}}$$

$$\nabla \bar{R}_\theta \approx \frac{1}{N}\sum_{n=1}^{N} R(\tau^n) \nabla logP(\tau^n|\theta) = \frac{1}{N}\sum_{n=1}^{N} R(\tau^n) \sum_{t=1}^{T_n} \nabla logp(a_t^n|s_t^n,\theta)$$

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It is very important t 這邊要注意的事情是 ive reward $R(\tau^n)$ of the whole trajectory τ^n instead of immediate reward r_t^n

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Gradient Ascent

$$\nabla \log P(\tau|\theta) = \sum_{t=1}^{T} \nabla \log p(a_t|s_t,\theta)$$

$$\theta^{new} \leftarrow \theta^{old} + \eta \nabla \bar{R}_{\theta^{old}}$$

$$\nabla \bar{R}_\theta \approx \frac{1}{N}\sum_{n=1}^{N} R(\tau^n) \nabla \log P(\tau^n|\theta) = \frac{1}{N}\sum_{n=1}^{N} R(\tau^n) \sum_{t=1}^{T_n} \nabla \log p(a_t^n|s_t^n,\theta)$$

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Why divided by $p(a_t^n|s_t^n,\theta)$?

e.g. in the sampling data …

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In τ^{13}, take action a

Gradient Ascent

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Why divided by $p(a_t^n|s_t^n,\theta)$?

e.g. in the sampling data … s has been seen in $\tau^{13}, \tau^{15}, \tau^{17}, \tau^{33}$

In τ^{13}, take action a

In τ^{17}, take action b

它採取 action a，在第 17 個它採取 b

Gradient Ascent

$$\nabla logP(\tau|\theta) = \sum_{t=1}^{T} \nabla logp(a_t|s_t,\theta)$$

$$\theta^{new} \leftarrow \theta^{old} + \eta \nabla \bar{R}_{\theta^{old}}$$

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它採取 action a，在第 17 個它採取 b

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e.g. in the sampling data … s has been seen in $\tau^{13}, \tau^{15}, \tau^{17}, \tau^{33}$

In τ^{13}, take action a

In τ^{15}, take action b

In τ^{17}, take action b

In τ^{33}, take action b

在 15 個採取 b

在 33 也採取 b

Gradient Ascent

$$\nabla \log P(\tau|\theta) = \sum_{t=1}^{T} \nabla \log p(a_t|s_t,\theta)$$

$$\theta^{new} \leftarrow \theta^{old} + \eta \nabla \bar{R}_{\theta^{old}}$$

$$\nabla \bar{R}_\theta \approx \frac{1}{N}\sum_{n=1}^{N} R(\tau^n)\nabla \log P(\tau^n|\theta) = \frac{1}{N}\sum_{n=1}^{N} R(\tau^n)\sum_{t=1}^{T_n} \nabla \log p(a_t^n|s_t^n,\theta)$$

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Why divided by $p(a_t^n|s_t^n,\theta)$?

e.g. in the sampling data … s has been seen in $\tau^{13}, \tau^{15}, \tau^{17}, \tau^{33}$

In τ^{13}, take action a	$R(\tau^{13}) = 2$	In τ^{15}, take action b
In τ^{17}, take action b	$R(\tau^{17}) = 1$	In τ^{33}, take action b

Gradient Ascent

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In τ^{13}, take action a	$R(\tau^{13}) = 2$	In τ^{15}, take action b	$R(\tau^{15}) = 1$
In τ^{17}, take action b	$R(\tau^{17}) = 1$	In τ^{33}, take action b	$R(\tau^{33}) = 1$

Add a Baseline

$$\theta^{new} \leftarrow \theta^{old} + \eta \nabla \bar{R}_{\theta^{old}}$$

$$\nabla \bar{R}_{\theta} \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla logp(a_t^n | s_t^n, \theta)$$

Add a Baseline

It is possible that $R(\tau^n)$ is always positive.

$$\theta^{new} \leftarrow \theta^{old} + \eta \nabla \bar{R}_{\theta^{old}}$$

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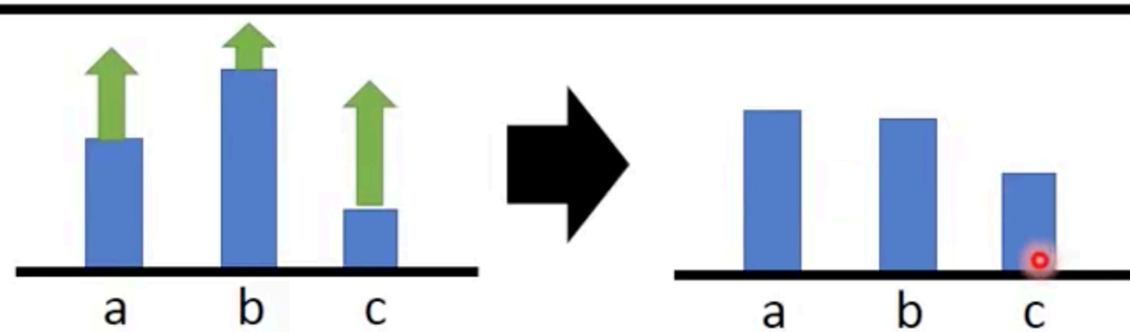
Ideal case



因為這邊是個機率，所以它會做 normalization

Add a Baseline

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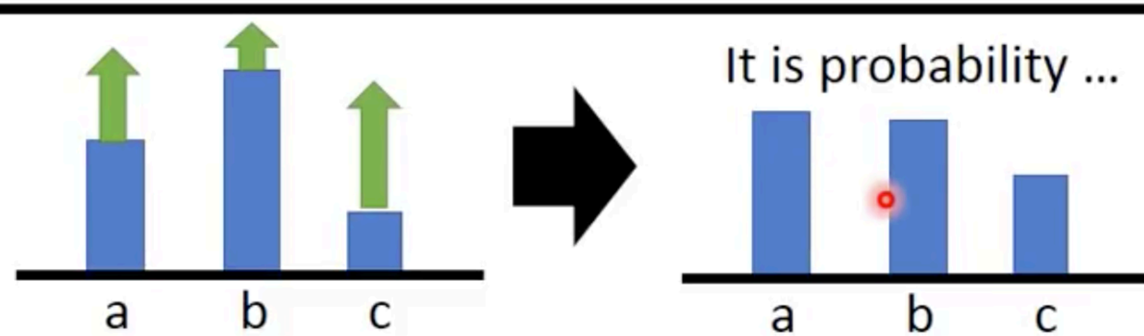
Ideal case



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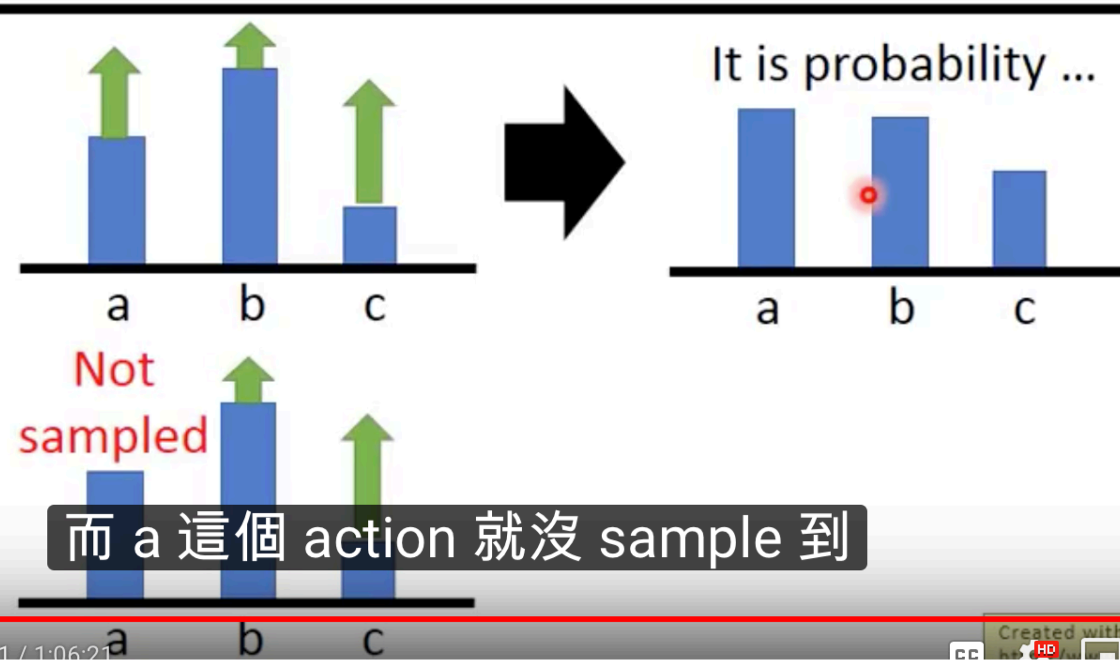
Ideal case



Sampling

......

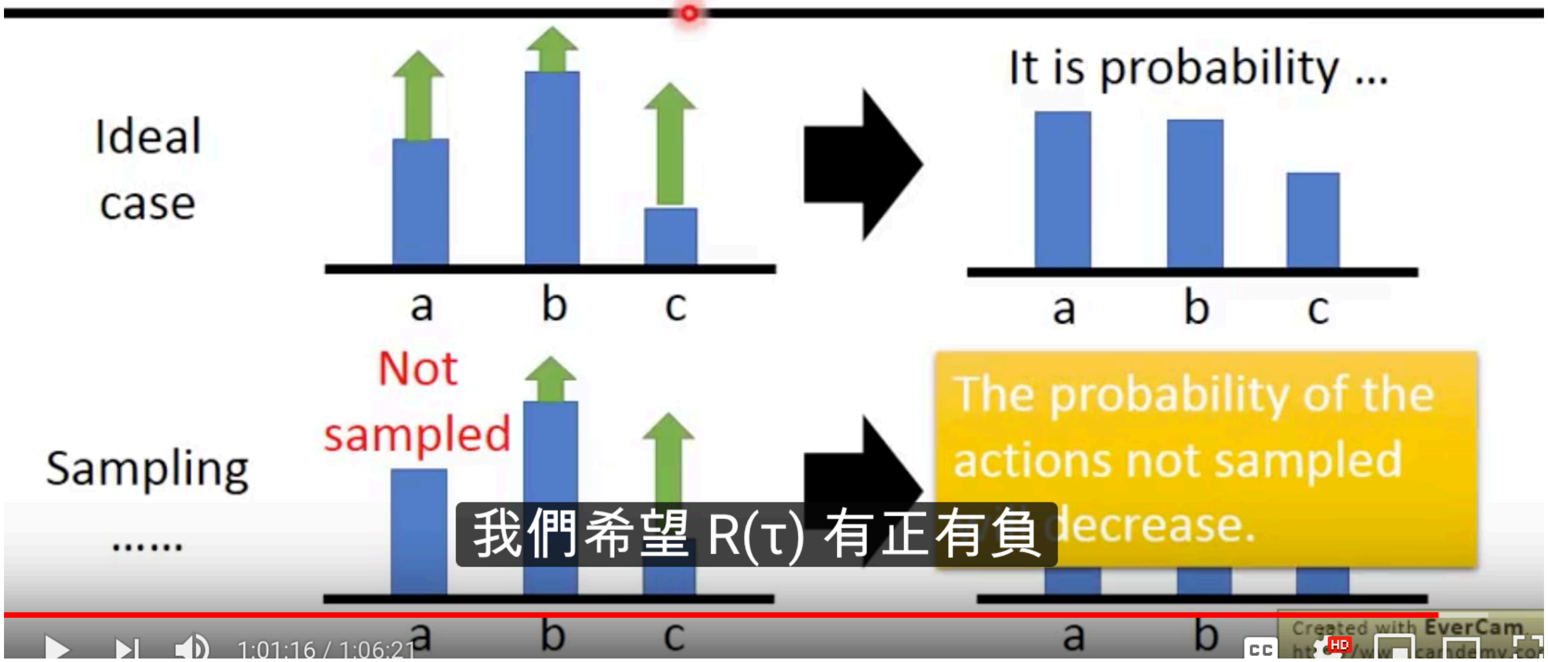
而 a 這個 action 就沒 sample 到

Add a Baseline

It is possible that $R(\tau^n)$ is always positive.

$$\theta^{new} \leftarrow \theta^{old} + \eta \nabla \bar{R}_{\theta^{old}}$$

$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla \log p(a_t^n | s_t^n, \theta)$$



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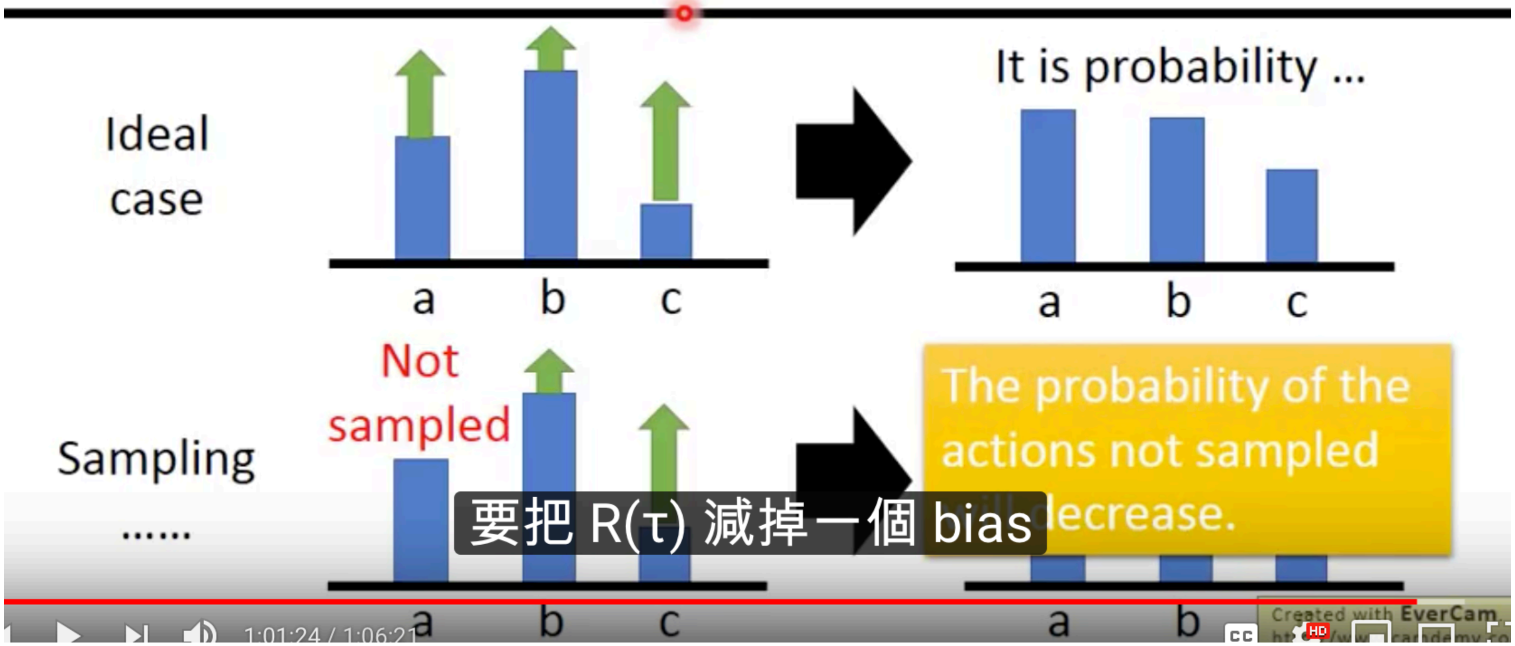
$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} \left(R(\tau^n) - b\right) \nabla \log p(a_t^n | s_t^n, \theta)$$



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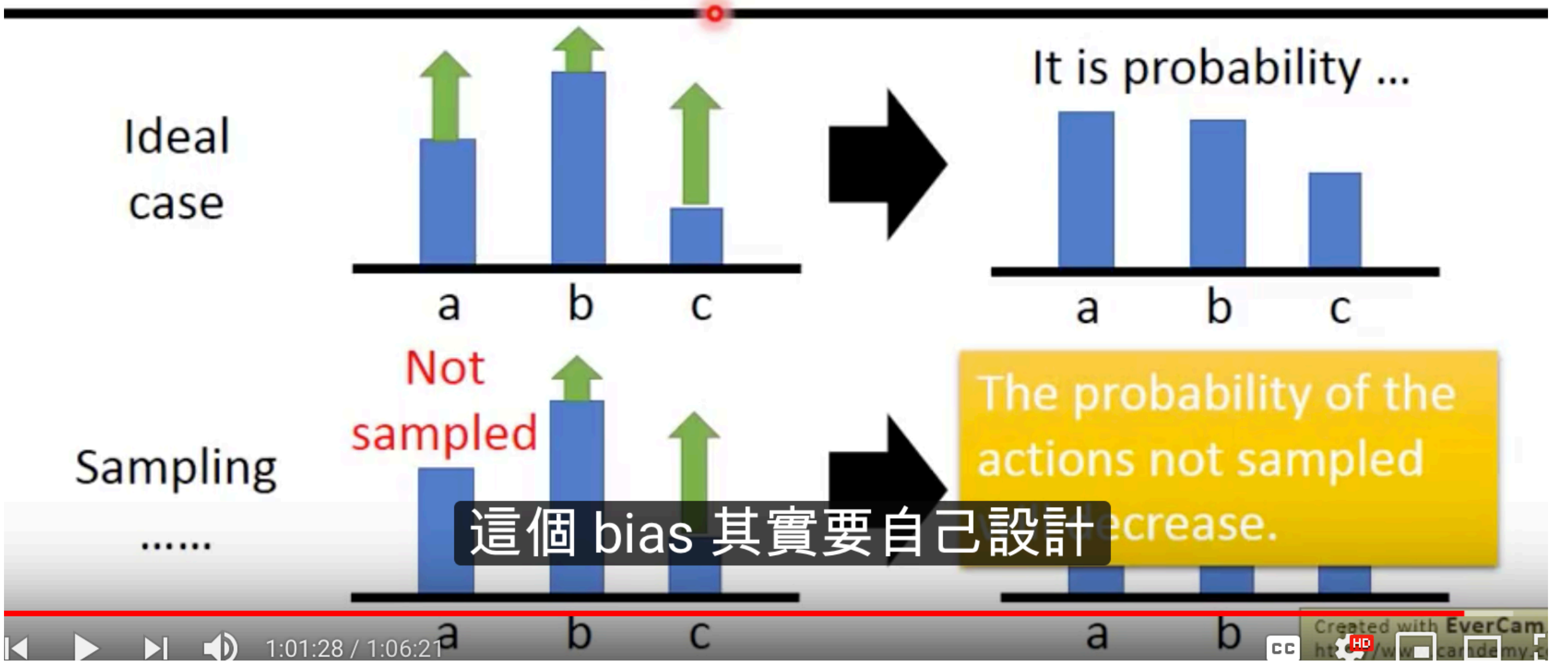
$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} (R(\tau^n) - b) \nabla \log p(a_t^n | s_t^n, \theta)$$



Add a Baseline

It is possible that $R(\tau^n)$ is always positive.

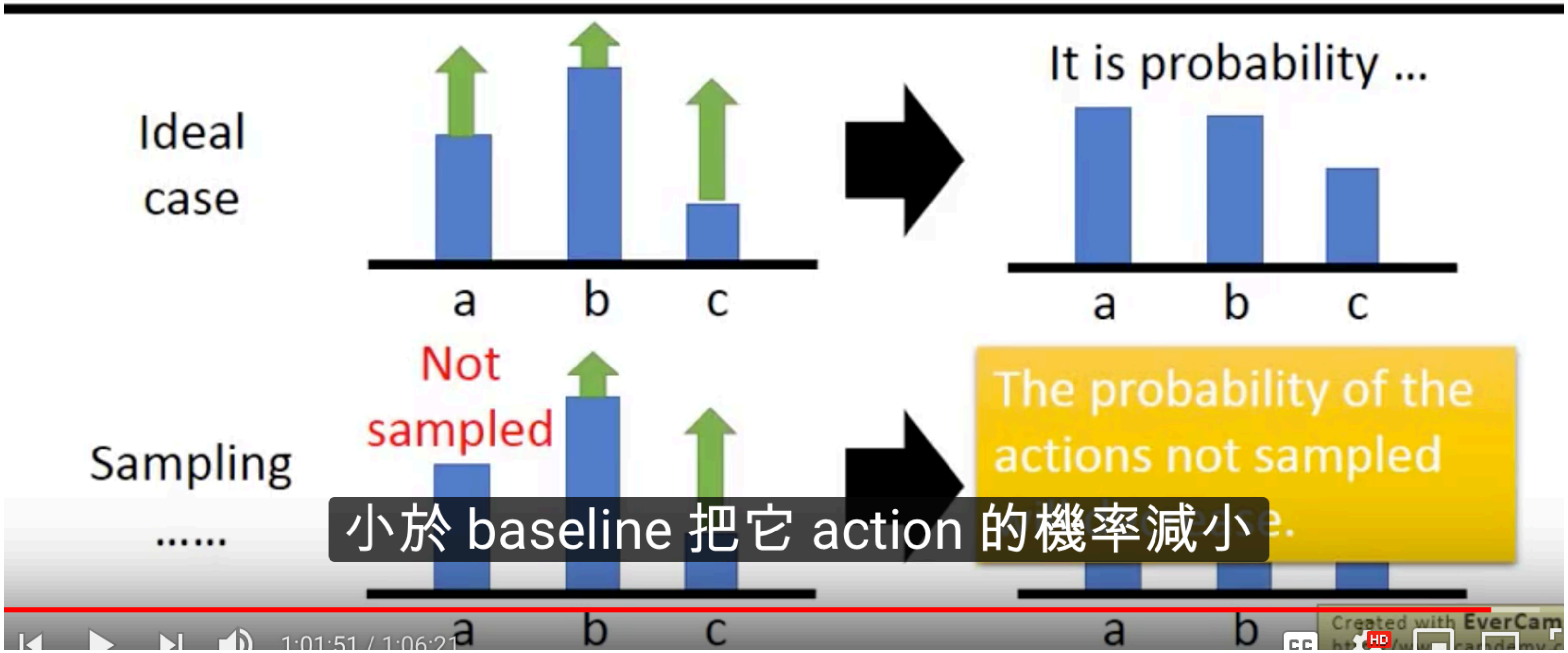
b: baseline

Reward better than baseline:
Increase probability

Reward worse than baseline:
Decrease probability

$$\theta^{new} \leftarrow \theta^{old} + \eta \nabla \bar{R}_{\theta^{old}}$$

$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} (R(\tau^n) - b) \nabla \log p(a_t^n | s_t^n, \theta)$$



Critic

- A critic does not determine the action.
- Given an actor, it evaluates the how good the actor is

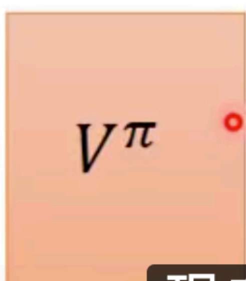
An actor can be found from a critic.

其實也可以從 Critic 得到一個 Actor

http://combiboilersleeds.com/picaso/critics/critics-4.html

Three kinds of Critics

- A critic is a function depending on the actor π it is evaluated
  - The function is represented by a neural network
- State value function $V^{\pi}(s)$
  - When using actor π, the *cumulated* reward expects to be obtained after seeing observation (state) s



這個 function 可以告訴你
現在看到某一個 observation 的時候

Three kinds of Critics

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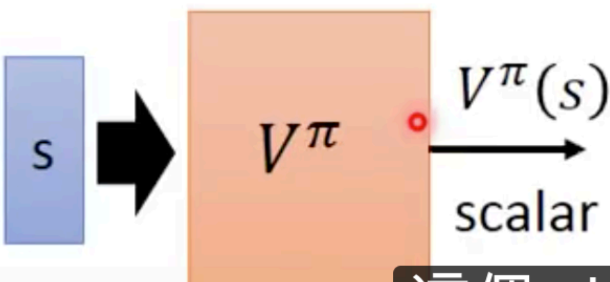

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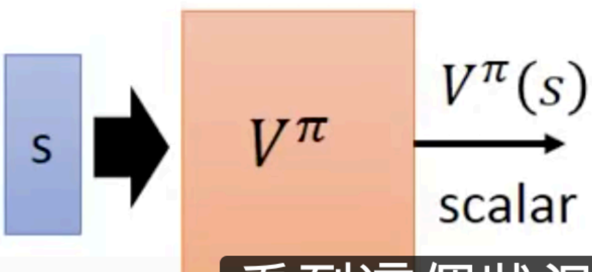
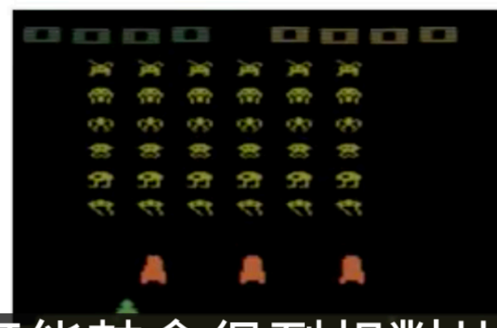




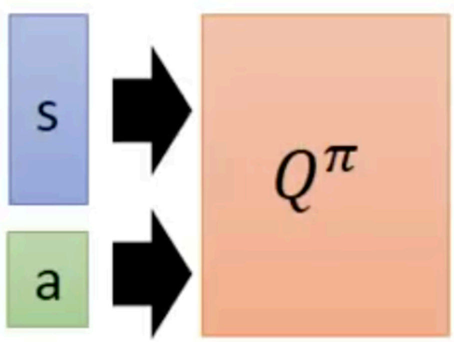
$V^{\pi}(s)$ is large

$V^{\pi}(s)$ is smaller

看到這個狀況可能較會得到相對比較少的值

Three kinds of Critics

- State-action value function $Q^{\pi}(s, a)$
  - When using actor π, the *cumulated* reward expects to be obtained after seeing observation s and taking a



Three kinds of Critics

- State-action value function $Q^\pi(s, a)$
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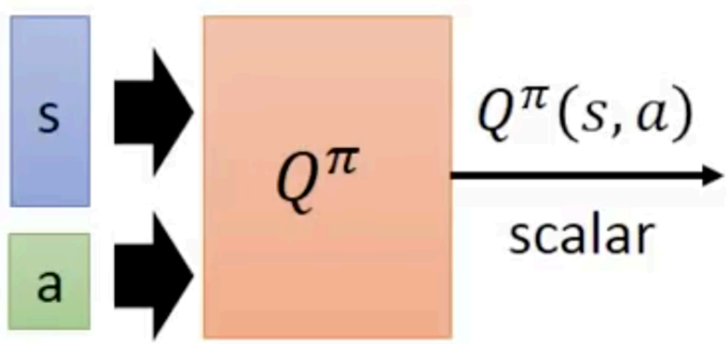
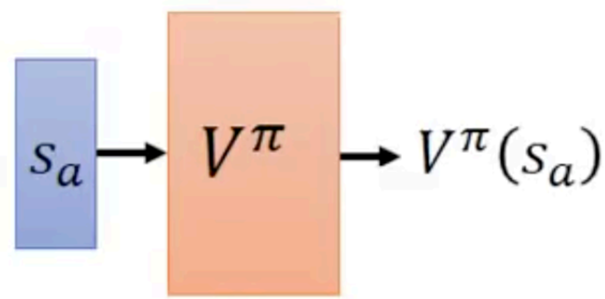


How to estimate $V^\pi(s)$

- Monte-Carlo based approach
  - The critic watches π playing the game

After seeing s_a,

Until the end of the episode, the cumulated reward is G_a



How to estimate $V^\pi(s)$

- Monte-Carlo based approach
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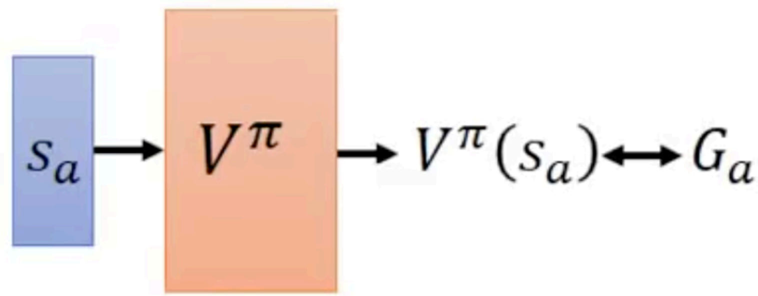
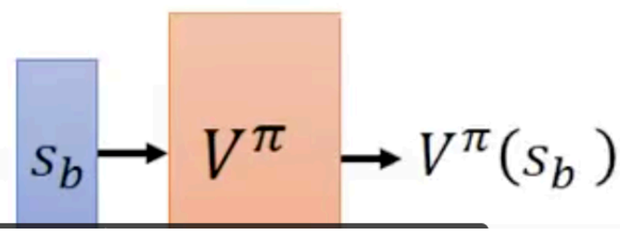
Until the end of the episode, the cumulated reward is G_a



After seeing s_b,

Until the end of the episode, the cumulated reward is G_b

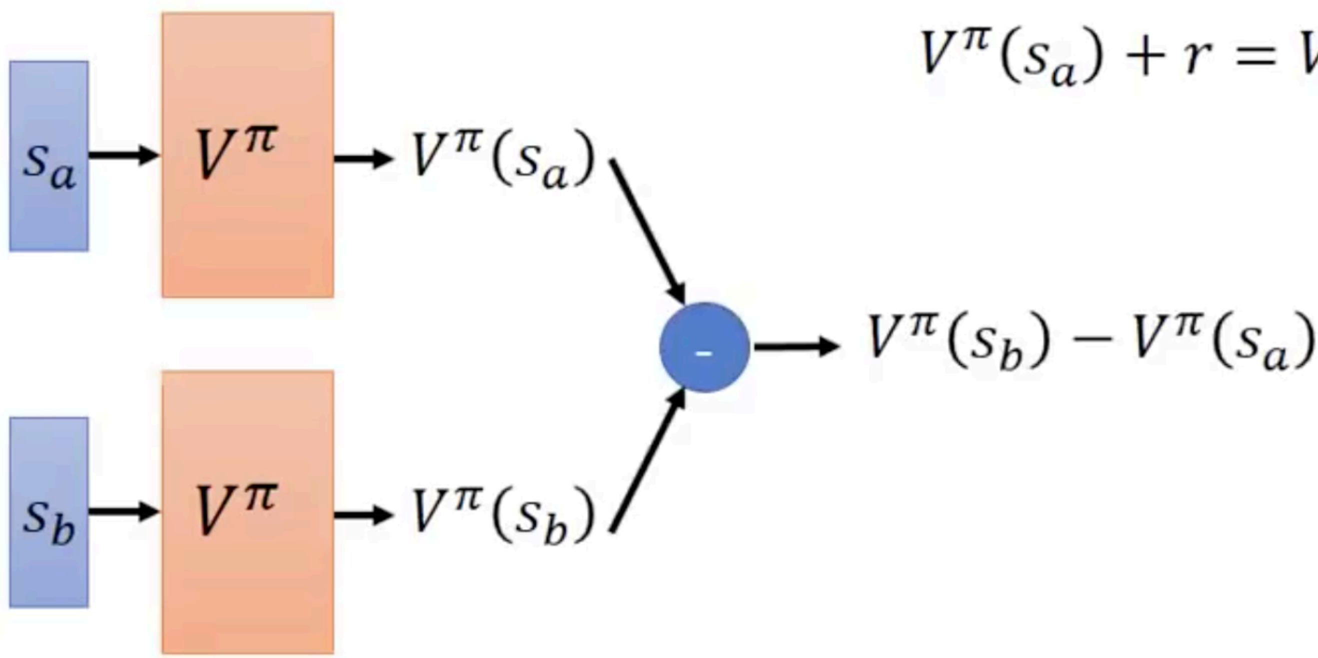


How to estimate $V^{\pi}(s)$

- Temporal-difference approach

$$\cdots s_a, a, r, s_b \cdots$$

$$V^{\pi}(s_a) + r = V^{\pi}(s_b)$$



Actor-Critic

$$\theta^{new} \leftarrow \theta^{old} + \eta \nabla \bar{R}_{\theta^{old}}$$

$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} \boxed{R(\tau^n)} \nabla log p(a_t^n | s_t^n, \theta)$$

Evaluated by critic

Advantage Function: $\underline{r_t^n} - \underline{\left(V^{\pi_\theta}(s_t^n) - V^{\pi_\theta}(s_{t+1}^n)\right)}$

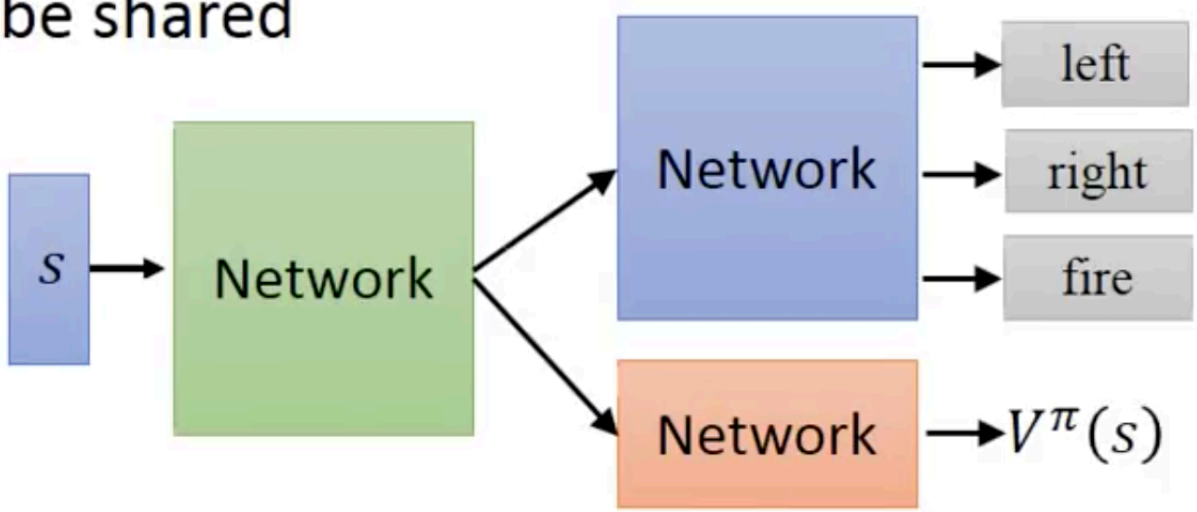
Baseline is added

The reward r_t^n we truly obtain when taking action a_t^n

Expected reward r_t^n we obtain if we use actor π_θ

Actor-Critic

- Tips
  - The parameters of actor $\pi(s)$ and critic $V^\pi(s)$ can be shared

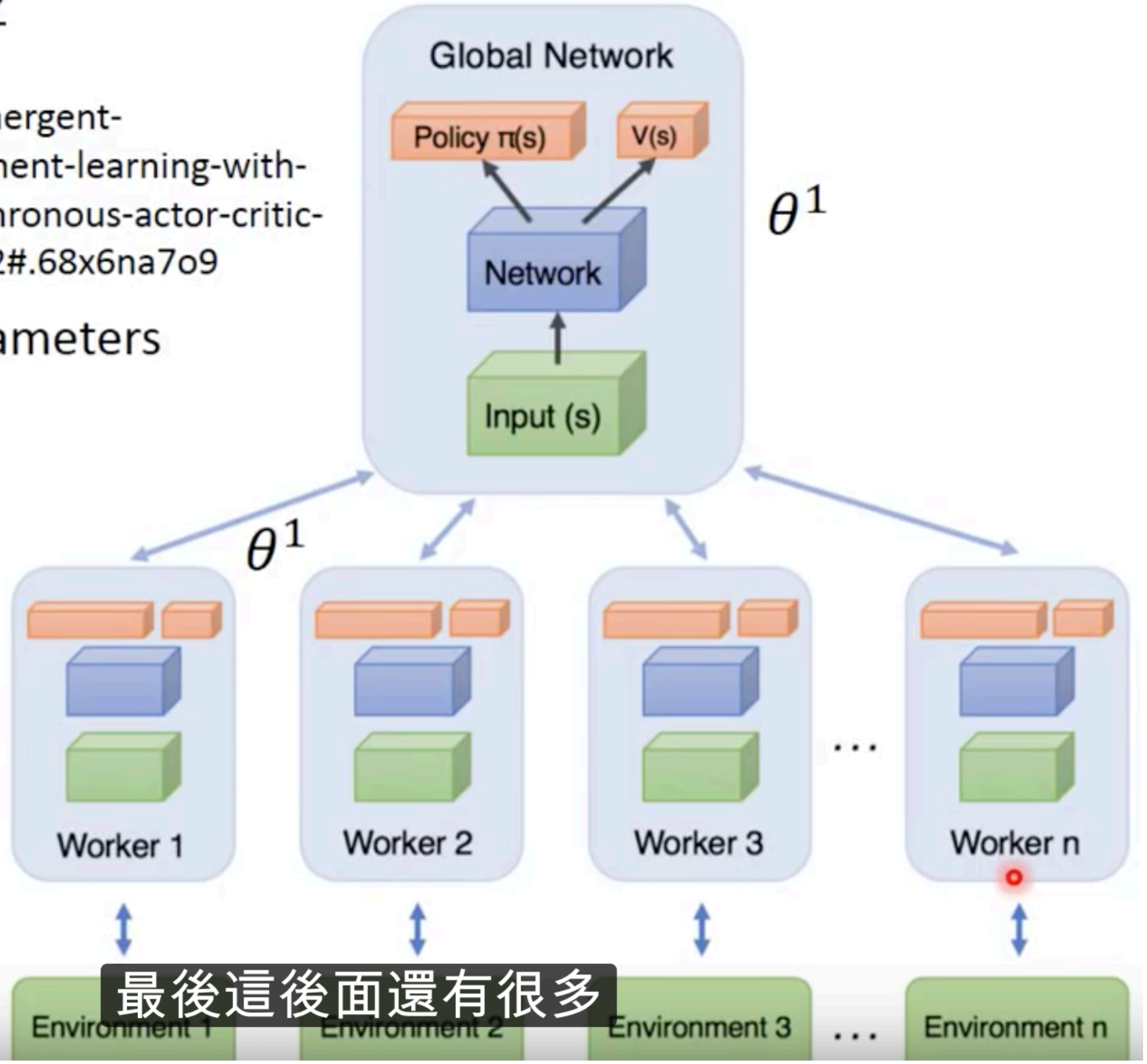


  - Use output entropy as regularization for $\pi(s)$

Asynchronous

Source of image: https://medium.com/emergent-future/simple-reinforcement-learning-with-tensorflow-part-8-asynchronous-actor-critic-agents-a3c-c88f72a5e9f2#.68x6na7o9

1. Copy global parameters



最後這後面還有很多