CSCE-411-HW-5

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29.4-1

The LP is :

$$\text{maximize: } 18x_1 + 12.5x_2$$

subject to:

$$x_1 + x_2 \leq 20$$
$$x_1 \leq 12$$
$$x_2 \leq 16$$
$$x_1, x_2 \geq 0$$

The dual LP is :

$$\text{minimize: } 20y_1 + 12y_2 + 16y_3$$

subject to:

$$y_1 + y_2 \geq 18$$
$$y_1 + y_3 \geq 12.5$$
$$y_1, y_2, y_3 \geq 0$$

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The slack form of the equation will be given by :

$$z = x_1 + 3x_2$$

$$x_3 = 8 - x_1 + x_2$$
$$x_4 = -3 + x_1 + x_2$$
$$x_5 = 2 + x_1 + 4x_2$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

The initial basic solution is *not feasible*, so form the auxiliary LP as follows

$$\text{maximize: } -x_0$$

subject to :

$$x_1 - x_2 - x_0 \leq 8$$
$$-x_1 - x_2 - x_0 \leq -3$$
$$-x_1 + 4x_2 - x_0 \leq 2$$
$$x_1, x_2, x_0 \geq 0$$

Now, we write this LP in slack form:

$$z = -x_0$$
$$x_3 = 8 - x_1 + x_2 + x_0$$
$$x_4 = -3 + x_1 + x_2 + x_0$$
$$x_5 = 2 + x_1 - 4x_2 + x_0$$
$$x_1, x_2, x_3, x_4, x_5, x_0 \geq 0$$

PIVOT *entering* : x_0, *leaving* : x_4

$$z = -3 + x_1 + x_2 - x_4$$
$$x_0 = 3 - x_1 - x_2 + x_4$$
$$x_3 = 11 - 2x_1 + x_4$$
$$x_5 = 5 - 5x_2 + x_4$$
$$x_1, x_2, x_3, x_4, x_5, x_0 \geq 0$$

The basic solution is feasible.

PIVOT *entering* : x_1, *leaving* : x_0

$$z = -x_0$$
$$x_1 = 3 - x_0 - x_2 + x_4$$
$$x_3 = 5 + 2x_0 + 2x_2 - x_4$$
$$x_5 = 5 - 5x_2 + x_4$$
$$x_1, x_2, x_3, x_4, x_5, x_0 \geq 0$$

The basic solution is now optimal for the auxiliary LP, so now we update the objective function in the parent LP after we set x_0 to 0 :

$$z = 3 + 2x_2 + x_4$$
$$x_1 = 3 - x_2 + x_4$$
$$x_3 = 5 + 2x_2 - x_4$$
$$x_5 = 5 - 5x_2 + x_4$$
$$x_1, x_2, x_3, x_4, x_5, x_0 \geq 0$$

PIVOT *entering* : x_2, *leaving* : x_5

$$z = 5 + \frac{7}{5}x_4 - \frac{2}{5}x_5$$
$$x_2 = 1 + \frac{1}{5}x_4 - \frac{1}{5}x_5$$
$$x_1 = 2 + \frac{4}{5}x_4 + \frac{1}{5}x_5$$
$$x_3 = 7 - \frac{3}{5}x_4 - \frac{2}{5}x_5$$
$$x_1, x_2, x_3, x_4, x_5, x_0 \geq 0$$

PIVOT *entering* : x_4, *leaving* : x_3

$$z = \frac{64}{3} - \frac{7}{3}x_3 - \frac{4}{3}x_5$$
$$x_4 = \frac{35}{3} - \frac{5}{3}x_3 - \frac{2}{3}x_5$$
$$x_2 = \frac{10}{3} - \frac{1}{3}x_3 - \frac{1}{3}x_5$$
$$x_1 = \frac{34}{3} - \frac{4}{3}x_3 - \frac{1}{3}x_5$$
$$x_1, x_2, x_3, x_4, x_5, x_0 \geq 0$$

The optimal solution is $(x_1, x_2) = (\frac{34}{3}, \frac{10}{3})$ and the optimal value of the objective function is $z = \frac{64}{3}$.

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The slack form of this equation will be given by :

$$z = x_1 - 2x_2$$

$$x_3 = 4 - x_1 - 2x_2$$

$$x_4 = -12 + 2x_1 + 6x_2$$

$$x_5 = 1 - x_2$$

The initial basic solution isn't feasible, so we will need to form the auxiliary linear program:

$$\text{maximize:} \ - x_0$$

subject to:

$$x_1 + 2x_2 - x_0 \leq 4$$

$$-2x_1 - 6x_2 - x_0 \leq -12$$

$$x_2 - x_0 \leq 1$$

$$x_1, x_2, x_0 \geq 0$$

The slack form is as follows

$$z = -x_0$$

$$x_3 = 4 - x_1 - 2x_2 + x_0$$

$$x_4 = -12 + 2x_1 + 6x_2 + x_0$$

$$x_5 = 1 - x_2 + x_0$$

PIVOT *entering:* x_0 and *leaving:* x_4

$$z = -12 + 2x_1 + 6x_2 - x4$$

$$x_3 = 16 - 3x_1 - 8x_2 + x_4$$

$$x_0 = 12 + x_4 - 2x_1 - 6x_2$$

$$x_5 = 13 - 2x_1 - 8x_2 + x_4$$

The basic solution is $(x_0, x_1, x_2, x_3, x_4, x_5) = (12, 0, 0, 16, 0, 13)$ which is feasible for the auxiliary LP.

PIVOT *entering:* x_1 and *leaving:* x_3

$$z = -\frac{4}{3} + \frac{2}{3}x_2 - \frac{2}{3}x_3 - \frac{1}{3}x_4$$
$$x_1 = \frac{16}{3} - \frac{8}{3}x_2 - \frac{1}{3}x_3 + \frac{1}{3}x_4$$
$$x_0 = \frac{4}{3} - \frac{2}{3}x_2 + \frac{2}{3}x_3 + \frac{1}{3}x_4$$
$$x_5 = \frac{7}{3} - \frac{8}{3}x_2 + \frac{2}{3}x_3 + \frac{1}{3}x_4$$

$(x_0, x_1, x_2, x_3, x_4, x_5) = (\frac{4}{3}, \frac{16}{3}, 0, 0, 0, \frac{7}{3})$ is the optimal solution for the auxiliary LP, and since $x_0 \neq 0$, the original LP is infeasible.