# CSCE 411 Design and Analysis of Algorithms

## HW7: Solutions

**Q - 29.3-5**

$$\begin{array}{rcl} \text{maximize} & & 18x_1 + 12.5x_2 \\ x_1 + x_2 & \leq & 20 \\ x_1 & \leq & 12 \\ x_2 & \leq & 16 \\ x_1, x_2 & \geq & 0 \end{array}$$

We first convert to slack form.

$$\begin{array}{rcl} z & = & 18x_1 + 12.5x_2 \\ x_3 & = & 20 - x_1 - x_2 \\ x_4 & = & 12 - x_1 \\ x_5 & = & 16 - x_2 \end{array}$$

The variable $x_1$ has a positive coefficient in the objective function. With respect to $x_1$, the second constraint is the most restrictive. Thus, we substitute $x_1$ using $x_1 = 12 - x_4$.

$$\begin{array}{rcl} z & = & 216 + 12.5x_2 - 18x_4 \\ x_3 & = & 8 - x_2 + x_4 \\ x_1 & = & 12 - x_4 \\ x_5 & = & 16 - x_2 \end{array}$$

The variable $x_2$ has a positive coefficient in the new objective function, and now the first constraint is the most restrictive. We replace it by $x_2 = 8 - x_3 + x_4$ and substitute $x_2$ everywhere else.

$$\begin{array}{rcl} z & = & 316 + 12.5x_3 - 5.5x_4 \\ x_2 & = & 8 - x_3 + x_4 \\ x_1 & = & 12 - x_4 \\ x_5 & = & 8 + x_3 - x_4 \end{array}$$

Now all coefficients are negative, so the basic solution, with $x_1 = 12$ and $x_2 = 8$, is optimal.

**Q - 29.3-6**

$$\begin{array}{rcl} \text{maximize} & & 5x_1 - 3x_2 \\ x_1 - x_2 & \leq & 1 \\ 2x_1 + x_2 & \leq & 12 \\ x_1, x_2 & \geq & 0 \end{array}$$

We first convert to slack form.

$$\begin{aligned} z &= 5x_1 - 3x_2 \\ x_3 &= 1 - x_1 + x_2 \\ x_4 &= 2 - 2x_1 - x_2 \end{aligned}$$

The nonbasic variables are $x_1$ and $x_2$. Of these, only $x_1$ has a positive coefficient in the objective function, so we must choose $x_e = x_1$. Both equations limit $x_1$ by 1, so we'll choose the first one to rewrite $x_1$ with. Using $x_1 = 1 - x_3 + x_2$ we obtain the new system

$$\begin{aligned} z &= 5 - 5x_3 + 2x_2 \\ x_1 &= 1 - x_3 + x_2 \\ x_4 &= 2x_3 - 2x_2 \end{aligned}$$

Now $x_2$ is the only nonbasic variable with positive coefficient in the objective function, so we set $x_e = x_2$. The last equation limits $x_2$ by 0 which is most restrictive, so we set $x_2 = x_3 - 0.5x_4$. Rewriting, our new system becomes

$$\begin{aligned} z &= 5 - 3x_3 - x_4 \\ x_1 &= 1 - 0.5x_4 \\ x_2 &= x_3 - 0.5x_4 \end{aligned}$$

Every nonbasic variable now has negative coefficient in the objective function, so we take the basic solution $(x_1, x_2, x_3, x_4) = (1, 0, 0, 0)$. The objective value this achieves is 5.