CSCE 411 Design and Analysis of Algorithms

HW3: Solutions

Q - 22.2-7

Idea:

We can create a graph G where each vertex represents a wrestler and each edge represent a rivalry. The graph will contain n vertices and r edges. Perform as many BFS's as needed to visit all vertices. Assign all wrestlers whose distance is even to the babyfaces and all wrestlers whose distance is odd to be heels. Then check each edge to verify that it goes between a babyface and a heel.

Pseudocode:

Input: Graph $G(V, E)$

Output: map M with key as node and value as designation

$M = \phi$
$current\_wristler = heels$
$Q = queue$
$Q.push(V[0])$
$M[V[0]] = babyface$
while Q is not empty do
    node $n = Q.pop()$
    $current\_wristler = \sim M[n]$
    for nodes(v) connected to n
        if $M[v] = \phi$ do
            $M[v] = current\_wristler$
        $Q.push(v)$

Correctness:

This is a modified form of BFS. BFS ensures the exhaustive search of all the nodes in the graph. For all the nodes, the map will ensures that if a node is not marked with any team, it should be marked with a team opposite to the adjacent node which in into consideration. Following this, all the node will be marked with a team.)

Time Complexity:

Time Complexity is $O(n + r)$

Q - 22.3-5

(a):

$\rightarrow$: Assume that edge (u, v) is a tree or forward edge. If (u, v) is a tree edge, then by the definition of tree edge v is first discovered by exploring edge (u, v). If (u, v) is a forward edge, then by the definition of forward edge v is an ancestor of u. In either case, d[u] $<$ d[v]. Since v was discovered after u, then must be finished prior to u being finished, hence f[v] $<$ f[u]. Every vertex must be discovered before it can be finished, so d[v] $<$ f[v]. Putting all three inequalities together yields d$[u]$ < d$[v]$ < f$[v]$ < f$[u]$

$\leftarrow$: Assume that for vertices u and v, d$[u]$ < d$[v]$ < f$[v]$ < f$[u]$. d$[v]$ < f$[v]$ can be established trivially. d$[u]$ < d$[v]$ and f$[v]$ < f$[u]$ imply that v was discovered after u, and finished before u. Therefore, edge (u, v) is a tree edge if v was discovered by traversing edge (u, v). Otherwise, (u, v) is a forward edge.

(b):

$\rightarrow$: Assume that edge (u, v) is a back edge. Then by definition, v is an ancestor of u, so d$[v]$ < d$[u]$. Since v was discovered before u, u will finish before v finishes. Hence, f$[u]$ < f$[v]$. d$[u]$ < f$[u]$ can be established trivially. Therefore d$[v]$ < d$[u]$ < f$[u]$ < f$[v]$.

$\leftarrow$: Assume d$[v]$ < d$[u]$ < f$[u]$ < f$[v]$. f$[u]$ < f$[v]$ and d$[v]$ < d$[u]$ imply that v was discovered before u and finished after u. Therefore v is an ancestor of u. Therefore (u, v) is a back edge.

(c):

$\rightarrow$: Assume that edge (u, v) is a cross edge. Therefore, there is no parental or ancestral relationship between u and v. d$[u]$ < f$[u]$ and d$[v]$ < f$[v]$ can be established trivially. If d$[u]$ < d$[v]$, then edge (u, v) would indicate a parental relationship between u and v in the depth-first tree, which cannot happen by the definition of cross edge. Hence, d$[u]$ > d$[v]$. Similarly, we cannot have d$[u]$ < f$[v]$, because this would indicate that v was finished (but not discovered) after u was discovered, but before u was finished, which cannot happen. Therefore, d$[v]$ < f$[v]$ < d$[u]$ < f$[u]$.

$\leftarrow$: Assume d$[v]$ < f$[v]$ < d$[u]$ < f$[u]$. Since f$[v]$ < d$[u]$, there is no parental or ancestral relationship between u and v. Therefore edge (u, v) is a cross edge.